\[
\Omega_{nm} = \frac{1}{\Omega_{mn}} e^{-\beta (E_n - E_m - W_{nm})}
\]

Work done by env. on sys during \( m \to n \)

\[
\dot{W} = \frac{1}{2} \sum_{nm} J_{nm}(+) W_{nm}
\]

\[
\dot{W} - T \dot{\mathbf{I}} = \dot{F} \text{ at all times}
\]
as \( t \to \infty \) because \( \dot{F} \to 0 \)

\[
( p_n(+) \to p_n^-)
\]

\( t \to \infty \): \( \dot{W} = T \dot{\mathbf{I}} \geq 0 \)

possible to have a NESS if \( \dot{W} > 0 \)

\[
\dot{W} = \dot{P}_{in} - \dot{P}_{out} = T \dot{\mathbf{I}}
\]

- net rate of work on sys (net power)
- input power (all > 0 terms)
- output power (all < 0 terms)
\[ P_{\text{out}} = P_{\text{in}} - TI \]
\[ \Rightarrow P_{\text{out}} = P_{\text{in}} - P_{\text{diss}} \]

to have a NESS
we need \( I > 0 \)
\[ \Rightarrow P_{\text{diss}} > 0 \]
always lose some input
power as heat
compare ESS: \( I = 0 \Rightarrow P_{\text{diss}} = 0 \)
but all \( J_{mn}(t) = 0 \) and
hence \( W = 0 = P_{\text{in}} = P_{\text{out}} \)

Example: light-sensitive protein

\[ E_{\text{in}} \xrightarrow{k_i} \text{ground state} \]
\[ \xrightarrow{r_1} \text{excited state} \]
\[ \xrightarrow{k_2} \text{excited state} \]
\[ \xrightarrow{k_3} \text{ground state} \]

\[ E_{\text{out}} \text{ work done by protein (chem. work)} \]

\[ E_1 \text{ (ground state)} \]
\[ E_2 \text{ (long-lived state)} \]
\[ E_3 \text{ (excited state)} \]

\[ \text{energy} \]

\[ \text{photon energy} \]

\[ \text{"work" or "signaling" state} \]
\[ r_1 = \text{spontaneous fluorescence} \]
\[ \text{emitting photon) typically fast } \sim [\text{ps-ns}]^{-1} \]
\[ k_2 = \text{rearrangement of protein lose some energy to heat} \]
\[ \text{must be fast to compete } w/ \ r, \]
\[ \frac{k_2}{r_1} \sim \text{"quantum yield" } \sim 0.1 - 0.7 \]
\[ \text{for most photoproteins} \]

**general det. balance relationships:**

\[ \frac{\Omega_{31}}{\Omega_{13}} = \frac{k_1}{r_1} = e^{-\beta (E_3 - E_1 - E_{in})} \]
\[ W_{31} = E_{in} \]

\[ \frac{\Omega_{23}}{\Omega_{32}} = \frac{k_2}{r_2} = e^{-\beta (E_2 - E_3)} \]

\[ \frac{\Omega_{12}}{\Omega_{21}} = \frac{k_5}{r_3} = e^{-\beta (E_1 - E_2 + E_{out})} \]
\[ W_{12} = -E_{out} \]
\text{master\ equ: } \frac{dp^n}{dt} = \sum_m J_{nm} \\
\text{stat. state} \\
\begin{align*}
0 &= \frac{dp_1^s}{dt} = J_{12}^s + J_{13}^s \Rightarrow J_{12}^s = -J_{13}^s = J_{31}^s \\
0 &= \frac{dp_2^s}{dt} = J_{23}^s + J_{21}^s \Rightarrow J_{23}^s = -J_{21}^s = J_{12}^s \\
0 &= \frac{dp_3^s}{dt} = J_{31}^s + J_{32}^s \Rightarrow J_{31}^s = -J_{32}^s = J_{23}^s \\
\end{align*}

J_{12} = J_{23} = J_{31} = J \quad \text{constant}

\begin{align*}
\text{equqs:} \\
J &= \Omega_{12} p_2^s - \Omega_{21} p_1^s \\
&= k_3 p_2^s - r_3 p_1^s \\
J &= k_1 p_1^s - r_1 p_3^s \\
J &= k_2 p_3^s - r_2 p_2^s \\
p_1^s + p_2^s + p_3^s &= 1
\end{align*}

4 \text{ equqs } \Rightarrow 4 \text{ unknowns}
\[(J, p_1^s, p_2^s, p_3^s)\]

**Solve**

\[J = \frac{k_1 k_2 k_3 - r_1 r_2 r_3}{D}\]

\[D = \text{func of } k_i + r_i > 0\]

\[J = \frac{k_1 k_2 k_3}{D} \left( 1 - \frac{r_1 r_2 r_3}{k_1 k_2 k_3} \right)\]

\[= \frac{k_1 k_2 k_3}{D} \left( 1 - e^{-\beta (E_{in} - E_{out})} \right)\]

so long as \(E_{in} > E_{out}\)

we have \(J > 0\)

\[\Rightarrow \text{NESS}\]