What about cells with flexible walls whose volume can change?

\[ N_{x} - n_{x} \]

\[ \Omega_{ji} \]

\[ n_{x} = \# \text{ of molec. of type } x \text{ inside cell} \]

\[ N_{x} - n_{x} = \text{"""" outside cell} \]

\[ x = 1, 2, \ldots, \# \text{ diff. types} \]

GB  focus on "impermecable molecules that can't leave on their own in reasonable amount of time"
\( g_i = \binom{N_{\text{pos}, \alpha}}{n_{\alpha}} \binom{N_{\text{pos}, \alpha}}{N_{\alpha} - n_{\alpha}} \sum \binom{m}{m} \)

\( N_{\text{pos}, \alpha} = \frac{V_i}{V_{\text{mol}, \alpha}} \) \hspace{1cm} \( N_{\text{pos}, \alpha} = \frac{V_{\text{tot}} - V_i}{V_{\text{mol}, \alpha}} \) \hspace{1cm} \text{for all diff. types}

vol. of type \( \alpha \) molec.

\[ \ln \binom{M}{m} \approx -m \ln \frac{m}{M} \]

\( M \gg 1, m \gg 1, \)

\( M \gg m \)

\( \Rightarrow \ln g_i \approx - \sum_{\alpha} \left[ n_{\alpha} \ln \frac{n_{\alpha} V_{\text{mol}, \alpha}}{V_i} \right. \]

\[ + (N_{\alpha} - n_{\alpha}) \ln \frac{(N_{\alpha} - n_{\alpha}) V_{\text{mol}, \alpha}}{V_{\text{tot}} - V_i} \]

\[ \text{recall:} \]

\[ \frac{\Omega_{j,i}}{\Omega_{i,j}} = e^{-\beta(E_j - E_i)} \Rightarrow \quad p_i^s = e^{-\beta E_i} \]

\[ \Rightarrow \quad \sum_{\text{ESS}} \]
Here:
\[
\frac{\Omega_{ji}}{\Omega_{ij}} = e^{-\beta (-k_B T \ln 6_j - (-k_B T \ln 6_i))}
\]

\[
\Rightarrow \quad p_i^s = e^{-\beta (-k_B T \ln 6_i)} \frac{\prod \ln 6_i}{Z} = e^{\ln 6_i} \frac{\prod \ln 6_i}{Z}
\]

System will go to equilibrium. The most likely state \(i\) (cell volume \(V_i\)) is the one with the largest \(\ln 6_i\):

to find max: \[
\frac{d}{dV_i} \ln 6_i = 0
\]

\[
\Rightarrow \quad \text{algebra} \quad \sum_{\alpha} \left[ -\frac{n_{\alpha}}{V_i} + \frac{(N_{\alpha} - n_{\alpha})}{V_{\text{tot}} - V_i} \right] C_{\alpha}^{\text{in}} + \left[ -\frac{n_{\alpha}}{V_i} + \frac{(N_{\alpha} - n_{\alpha})}{V_{\text{tot}} - V_i} \right] C_{\alpha}^{\text{out}} = 0
\]
\[ \sum_{\alpha} C^{{in}}_{\alpha} = \sum_{\alpha} C^{{out}}_{\alpha} \]

total conc's of impermeable particles are balanced inside & out in equilibrium

\[ \Rightarrow \]

water flows in as system equilibrates

\[ \Rightarrow \]

osmotic stress

danger: cells can't expand or shrink indefinitely

\[ \Rightarrow \] end up w/ a breakdown of cell membrane!

All cells w/ flexible membranes thus must have transporters or other mechanisms to regulate osmotic stress.
Key lessons for life:

- Need containers $\Rightarrow$ we need to concentrate in order for reactions to occur quickly

$\Rightarrow$ need imbalances of various molecular concs (i.e. ATP vs ADP + P) to drive biochemical systems out of equilibrium

- Need transport mechanisms (ATP-driven proteins) to regulate osmotic stress