Look at ions on either side of membranes & also transport of ions.

Starting point: in real cells, many molecular types are charged but there is an approximate balance between total positive & negative charge both inside & outside cell

⇒ electro-neutrality

At typical densities, there are usually a couple of nm on avg. b/t ions
Pair of $\oplus$ and $\ominus$ ions,

Coulomb energy of attraction

$$= \frac{-e^2}{4\pi \varepsilon_r r}$$

$r$ = separation

$\varepsilon_r$ = permittivity of surrounding material

$\varepsilon_0$ = vacuum permittivity

Relative permittivity

air $\varepsilon_r \approx 1$

water $\varepsilon_r \approx 80$

\(1\) nm

in water, Coulomb energy $\approx -0.7 k_B T$

weak attraction easily broken up by thermal fluctuations

(reality: even weaker because of additional screening from other ions)

$\implies$ net effect: ions will be jostled by thermal fluctuations
Mix and diffuse throughout the cell. Imagine putting one extra negative charge inside the cell, leaving one extra positive charge outside.

Zoom in:

- 4 nm lipid bilayer
- $\varepsilon_r$ inside membrane $\varepsilon_r \approx 2$

Coulomb energy:

$$\sim -\frac{e^2}{4\pi\varepsilon_r\varepsilon_0 r} = 0.2 \text{ eV}$$

$$\frac{8.8 \times 10^{-12}}{4 \text{ nm}} \approx 8 \text{ k}_B T$$

Quite strong attraction (stable against thermal fluctuations).
If we have many such excess charges they will build uniformly at either surface of membrane

⇒ you end up with a capacitor (locally looks like a parallel plate capacitor)

capacitor: \( \rho = \frac{Q}{A} = \frac{\text{charge}}{\text{unit area}} \)

voltage drop \( V = \frac{\rho d}{\varepsilon} \)

capacitance \( C = \frac{Q}{V} = \frac{\varepsilon A}{d} \)

specific capacitance (per unit area) \( \frac{C}{A} = \frac{\varepsilon}{d} \sim 1 \text{ MF/cm}^2 \)

typical cells membrane potentials have a narrow range:
\[ V \sim -50 \leftrightarrow -200 \text{ mV} \]
(measured from outside to in)

\[ \begin{array}{c c c c}
& + & & - \\
out & \rightarrow & & in \\
& & + & \\
& \rightarrow & & - \\
\end{array} \]

\[ V \sim -100 \text{ mV} \]

\[ \# \text{ charges on surface} \]

\[ \frac{Q}{e} = \frac{CV}{e} = 2 \times 10^4 \text{ charges} \]

\[ A \sim 3 \mu m^2 \text{ on surface} \]

Total ion concentration \( \sim 100 \text{ mM} \)

\( \sim 10^8 \text{ charges in a fL volume of cell} \)

(bacterial volume \( \sim fL \))