detailed balance condition:

\[
\frac{W_+}{W_-} = e^{-\beta(E_{out} - E_{in} + k_BT \ln \frac{c_{out}}{c_{in}})}
\]

\[
= e^{-\beta \left( -q\Delta V + k_BT \ln \frac{c_{out}}{c_{in}} \right)}
\]

\[
\approx \sqrt{\frac{M_{out} - M_{in}}{M_{out} + M_{in}}}
\]

potential

\[
\Delta V = V_{in} - V_{out}
\]

\[
\approx -100 \text{ mV}
\]

\[
e^{\Delta V} \approx 4 k_BT
\]

K^+ ions: \( c_{in} = 400 \text{ mM} \quad c_{out} = 20 \text{ mM} \)

\(\Rightarrow\) Note: when the following is true

\[-q\Delta V + k_BT \ln \frac{c_{out}}{c_{in}} = 0 \Rightarrow W_+ = W_-\]

this occurs when \( \Delta V = \frac{k_BT}{q} \ln \frac{c_{out}}{c_{in}} \equiv V_N \)
\[ \Rightarrow \text{Nernst potential} \]
(Note: diff. Nernst potential for each ion type b/c depends on \( C_{out}, C_{in}, q \))

\[ \text{K}^+ \text{ ions: } q = +e \Rightarrow V_N = -75 \text{ mV} \]

\[ \begin{align*}
\text{K}^+ & \quad C_{out} = 20 \text{ mM} \\
\text{K}^+ & \quad C_{in} = 400 \text{ mM} \\
\Rightarrow & \quad 10^8 \text{ ions/\mu L}
\end{align*} \]

\[ \sim 10^4 \text{ excess charges to get potential} \]
\[ \Delta V = -75 \text{ mV} = V_N \Rightarrow W_+ = W_- \]

\[ \begin{align*}
\text{K}^+ & \quad \text{net flow of} \\
\text{K}^+ & \quad \text{ions thru membrane} = 0
\end{align*} \]

If \( \Delta V > V_N \Rightarrow W_+ > W_- \quad \text{net flow out} \]
\[ \Delta V < V_N \Rightarrow W_- < W_+ \quad \text{net flow in} \]

Imagine starting with \( \Delta V > V_N \)

\[ \Rightarrow \text{flow out leads to more + excess charges outside} \]
\[ \Rightarrow \text{decrease in } \Delta V \ (C_{in} + C_{out} \text{ stay roughly same}) \]
Eventually $\Delta V$ reaches $V_N$
* flow stops

Analogously if $\Delta V < V_N$
* system eventually increases $\Delta V$ thru inward flow
* $\Delta V \rightarrow V_N$ & the flow stops

* for transport of a single ion type thru channels on membrane
* $V_N$ is a stable equilibrium point for membrane potential

Net current out of cell:

$$I = 6 (\Delta V - V_N) + \text{higher order terms}$$

Taylor expand around $V_N$

$I > 0$ : current out of cell
$I < 0$ : current into the cell
\[
\frac{\text{current}}{\text{area}} = j = \frac{I}{A} = g (\Delta V - V_N)
\]

\[\Rightarrow \frac{6}{A} \approx 10 \Omega^{-1} \text{m}^{-2} \text{ for } K^+ \text{ channels}
\]

 Reality is more complicated:

- multiple ion types, each with own channels

\[
\frac{\text{current}}{\text{area}} \text{ for type } i \quad j_i = g_i (\Delta V - V_N^{(i)})
\]

\[V_N^{(i)} = \frac{k_B T}{q_i} \ln \frac{C_{out}^{(i)}}{C_{in}^{(i)}}
\]

most important \( i = \text{Na}^+, K^+ \)

\[q_i = e \]
Na\(^+\): \(50\text{mM}\)  
K\(^+\): \(400\text{mM}\)  
\(V_{N}^{in}\): \(54\text{mV}\)  
\(V_{N}^{out}\): \(-75\text{mV}\)  

How do we maintain equilibrium in this scenario? Can we get all ion currents = 0? \(\Delta V = V_{o}\) equilibrium potential

\[
\dot{J}_{Na} = g_{Na} (V_{o} - V_{N}^{Na}) + \dot{J}_{Na}^{\text{pump}} = 0
\]

\[
\dot{J}_{K} = g_{K} (V_{o} - V_{N}^{K}) + \dot{J}_{K}^{\text{pump}} = 0
\]

Actual value of \(V_{o} = -65\text{mV}\)

In order to maintain equilibrium at \(V_{o}\) \(\Rightarrow\) must have a mechanism to pump Na\(^+\) out + K\(^+\) in.

Job done by Na/K pump: takes 3 Na\(^+\) out + 2 K\(^+\) in for every cycle, consuming one ATP.

Neurons/kidney cells: 50-70% of total cell energy expended on these Na/K pumps.
Note brain alone accounts of total bodily ATP consumption.

Kidney ~ 10% of total bodily ATP consumption (even though 0.5% of total mass)