cell membrane:

\[ \Delta V = V_{in} - V_{out} \]

at \( \Delta V = V_o \approx -65 \text{ mV} \)

all ion currents = 0

(resting potential)

\[ \Rightarrow 54 \text{ mV} \]

\[ \dot{J}_Na = g_{Na} (V_o - V_{N}^{Na}) + \dot{J}_{Na}^{pump} = 0 \quad (1) \]

\[ \dot{J}_K = g_K (V_o - V_{N}^{K}) + \dot{J}_{K}^{pump} = 0 \quad (2) \]

\[ \Rightarrow -75 \text{ mV} \]

we know b/c of the nature of the pump that:

\[ \dot{J}_{K}^{pump} = -\frac{2}{3} \dot{J}_{Na}^{pump} \quad (3) \]

take (1), (2), (3) + solve for \( V_o \).
\[ g_k (V_o - V_{N}^k) = - \int_{K}^{\text{pump}} \]
\[ = \frac{2}{3} j_{Na}^{\text{pump}} = - \frac{2}{3} g_{Na} (V_o - V_{Na}^N) \]

\[ \Rightarrow V_o = \frac{2 g_{Na} V_{Na}^N + 3 g_k V_{N}^k}{2 g_{Na} + 3 g_k} \]

In normal resting state, most Na channels are closed:

\[ g_k \gg g_{Na} : \quad g_k = 25 g_{Na} \quad \text{for squid} \]

\[ \Rightarrow V_o \text{ should be close to } V_{N}^k \]

In fact \( V_o = -65 \text{ mV} \) which is close to \( V_{N}^k = -75 \text{ mV} \)

What happens when \( \Delta V \neq V_o \)?

\[ \Delta V = V_o + u \quad \text{where } u \neq 0 \]
\[
\dot{J}_{\text{tot}} = \dot{J}_K + \dot{J}_{\text{Na}} = g_K (u + V_o - V_N^K) + j_{\text{pump}}^K + g_{\text{Na}} (u + V_o - V_N^{\text{Na}}) + j_{\text{pump}}^\text{Na} = (g_K + g_{\text{Na}}) u + g_{\text{tot}} (\Delta V - V_o)
\]

- If \( \Delta V < V_o \)
  - \( \dot{J}_{\text{tot}} < 0 \)
  - \( \Delta V \) becomes more positive
  - \( \Delta V \rightarrow V_o \) from below

- If \( \Delta V > V_o \)
  - \( \dot{J}_{\text{tot}} > 0 \)
  - \( \Delta V \) becomes more negative
  - \( \Delta V \rightarrow V_o \) from above
  - Eventually return to resting state

What if Na channels all opened?

\[ g_{\text{Na}} \rightarrow \tilde{g}_{\text{Na}} \gg g_K \]

\[ V_o \rightarrow \tilde{V}_o = 40 \text{ mV} \] close to \( V_N^{\text{Na}} \)
Turns out $g_{Na}$ is actually a function of $\Delta V$:

Concrete example:
neurons after death

$\rho I$

$K^+$ ions

$\rho \sim 0.5 \text{ mm} \quad \text{squid}$

$\text{rat}$
start hear $V_0$ + 
then current modifies $\Delta V$:

$$\dot{V}_{\text{tot}} = g_{\text{tot}} (V_0 - V_N^k) + \text{no pumping}$$

starting point $-65 \rightarrow (-75)$

$> 0$

$$\dot{J}_{\text{tot}} A_{\text{mem}} \approx 2 \times 10^7 \text{ /s}$$

rat: 40 secs to leak out
squid: $10^5$ sec to leak out

as leak occurs $\Rightarrow$ concentration $C_{\text{in}}$ of $K^+$ decreases forcing $V_N^k$ to be less negative

$\Rightarrow$ forces $V_0$ up, eventually opening up Na channels
Result: odd “wave of death” of action potentials firing all nearly at same time

Can achieve similar result pouring soy sauce on squid shortly after death ⇒ high Con of Na⁺ drives up $V_{Na}^{\text{Na}}$, driving up $V_o$ & opening up Na channels (see video of “dancing” squid sushi)