Break problem into three parts:

1) how long on avg to leave initial state

2) which state do you visit after leaving $i$? (probability)

$\tau_i =$ avg. time it takes to get from $i$ to $i_c$ for first time
3) recursive argument \( \Rightarrow \) use results of 1) + 2) to build an equation for \( T_i \)

**Element #1**

\[
\frac{.3}{.3 + .1} = 0.75
\]

\( \Delta t = 1s \)

\( w_1 \Delta t = 0.05 \)

\( w_2 \Delta t = 10^{-20} \)

\( n \Delta t \equiv t + t + \Delta t \)

prob. of not leaving \( i = 1 - (w_1 + w_2) \Delta t \)

prob. that you leave state \( i \) exactly between time \( t + t + \Delta t \) \( \equiv f_i(t) \Delta t \)

note: \( \int_0^\infty dt f_i(t) = 1 \)
\[ f_i(t) \delta t = \left[ 1 - (w_1 + w_2) \delta t \right]^n \left( w_1 + w_2 \right) \delta t \]

Prob. survive \( n \) time steps \( \Rightarrow \) prob. you die in next time step

In general,

\[ \text{prob. of leaving } i \text{ in time step } \delta t = \sum_{j \neq i} \Omega_{ji} \delta t = |\Omega_{ii}| \delta t \]

Recall: \( \Omega_{ii} = -\sum_{j \neq i} \Omega_{ji} \) (b/c columns sum to zero)

\[ \text{prob. of not leaving } i \quad \Rightarrow \quad 1 - |\Omega_{ii}| \delta t \]

\[ \delta t f_i(t) = \left[ 1 - |\Omega_{ii}| \delta t \right]^n |\Omega_{ii}| \delta t \]

\[ \delta t = \frac{t}{n} \]

\[ \Rightarrow \quad \delta t f_i(t) = \left( 1 - |\Omega_{ii}| \frac{t}{n} \right)^n |\Omega_{ii}| \delta t \]

Small \( \delta t \Rightarrow \) large \( n \)
\[ f_i(t) = \exp(-1 \Omega_{ii} t) \Omega_{ii} \]

avg. escape time

\[ \overline{t}_i^{esc} = \int_0^{\infty} dt \, t \, f_i(t) = \frac{1}{|\Omega_{ii}|} \]

i.e. \[ |\Omega_{ii}| = 3 \, \text{s}^{-1} \]

\[ \overline{t}_i^{esc} = \frac{1}{3} \, \text{s} \]

2) After escaping, which state did we end up in?

frog problem: prob. of dying via truck

\[ = \frac{W_1 \delta t}{W_1 \delta t + W_2 \delta t} = \frac{W_1}{W_1 + W_2} \]
Prob. of meteor death = \( \frac{W_2}{W_1 + W_2} \)

In general \( \Pi_{ji} = \frac{\Omega_{ji}}{\Omega_{ii}} \)

\[ \downarrow \]

Prob. of going to \( j \) after leaving \( i \)

\[ \tau_{esc} = \frac{1}{W_2} \approx 10^{20} \text{ s} \]

\[ \tau_{esc} = \frac{1}{W_1 + W_2} \approx 20 \text{ s} \]

Regardless of cause of death
note: probabilities of transition $\Omega_{ji}$ are time-independent