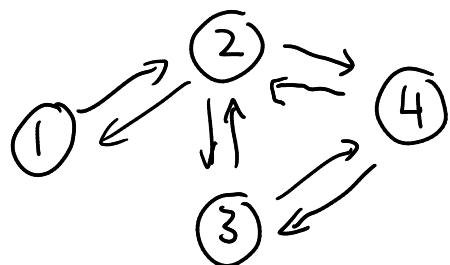


Assumption:  $P_n(t) \rightarrow P_n^s$

Can we prove this? Under what conditions?

Proof: ingredients:



- any system w/ a time-indep. matrix of trans.

$$\Omega$$

- graph for  $\Omega$  is connected

Recall some quantum mechanics:

	quantum	master equ.	elements: $P_n(t)$
description of state	$ \psi(t)\rangle$ vector in Hilbert space	$\vec{p}(t)$ vector in prob. space	
dynamical eqn.	$i\hbar \frac{\partial}{\partial t}  \psi(t)\rangle = \hat{H}  \psi(t)\rangle$	$\frac{\partial}{\partial t} \vec{p}(t) = \Omega \vec{p}(t)$	
solution	$ \psi(t)\rangle = \underbrace{e^{i\hat{H}t/\hbar}}_{\text{propagator } \hat{U}(t)}  \psi(0)\rangle$	$\vec{p}(t) = e^{-\Omega t} \vec{p}(0)$	
avg. of observable	$A(t) = \langle \psi(t)   \hat{A}   \psi(t) \rangle = \langle \psi(0)   \hat{A}^H(t)   \psi(0) \rangle$	$A(t) = \vec{A} \cdot \vec{p}(t) = \vec{A}^H(t) \cdot \vec{p}(0)$	$\vec{A}^H = \vec{A}^T e^{-\Omega t}$

claim:  $\vec{p}(t) = e^{-\Omega t} \vec{p}(0)$  solves master equation

$$e^{-\Omega t} \equiv \mathbb{I} + \Omega t + \frac{\Omega^2 t^2}{2!} + \frac{\Omega^3 t^3}{3!} + \dots$$

identity

$$\begin{aligned}
 \frac{d}{dt} e^{-\Omega t} &= 0 + \Omega + \Omega^2 t + \frac{\Omega^3 t^2}{2!} + \dots \\
 &= \Omega \left( \mathbb{I} + \Omega t + \frac{\Omega^2 t^2}{2!} + \dots \right)
 \end{aligned}$$

$$= \Omega e^{\Omega t} = e^{\Omega t} \Omega$$

$$\frac{d}{dt} \left( \underbrace{e^{\Omega t} \vec{p}(0)}_{\vec{p}(t)} \right) = \Omega \underbrace{e^{\Omega t} \vec{p}(0)}_{\vec{p}(t)}$$

master eqn.  
is satisfied!

averages:  
(state  
func's)

$$A(t) = \sum_n A_n p_n(t)$$

$$= \vec{A} \cdot \vec{p}(t)$$

define  $\vec{A}$   
whose nth  
comp. is  $A_n$

recall: Heisenberg picture

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle$$

$$A(t) = \langle \psi(t) | \hat{A} | \psi(t) \rangle$$

$$= \langle \psi(0) | \underbrace{\hat{U}^\dagger(t) \hat{A} \hat{U}(t)}_{\equiv \hat{A}^H(t)} | \psi(0) \rangle$$

Heisenberg operator

"classical" Heisenberg:

$$A(t) = \sum_n A_n p_n(t) \quad \vec{p}(t) = e^{\Omega t} \vec{p}(0)$$

$$= \sum_n A_n (e^{\Omega t} \vec{p}(0))_n$$

$$= \sum_{n,m} A_n (e^{\Omega t})_{nm} p_m(0)$$

$$= \sum_m A_m^H(t) p_m(0)$$

where:  $A_m^H(t) \equiv \sum_n A_n (e^{\Omega t})_{nm}$

$$\Rightarrow \vec{A}^H(t) = \vec{A}^T e^{\Omega t}$$

$$\Rightarrow A(t) = \vec{A}^H(t) \cdot \vec{p}(0)$$

	quantum	master equ.	elements:
description of state	$ \psi(t)\rangle$ vector in Hilbert space	$\vec{p}(t)$ vector in prob. space	$p_n(t)$
dynamical equ.	$i\hbar \frac{\partial}{\partial t}  \psi(t)\rangle = \hat{H}  \psi(t)\rangle$	$\frac{\partial}{\partial t} \vec{p}(t) = \Omega \vec{p}(t)$	
solution	$ \psi(t)\rangle = \underbrace{e^{i\hat{H}t/\hbar}}_{\text{propagator } \hat{U}(t)}  \psi(0)\rangle$	$\vec{p}(t) = e^{\Omega t} \vec{p}(0)$	
avg. of observable	$A(t) = \langle \psi(t)   \hat{A}   \psi(t) \rangle = \langle \psi(0)   \hat{A}^H(t)   \psi(0) \rangle$	$A(t) = \vec{A} \cdot \vec{p}(t) = \vec{A}^H(t) \cdot \vec{p}(0)$	$\vec{A}^H = \vec{A}^T e^{\Omega t}$
Heis. dynamical equation	$\frac{d}{dt} \vec{A}^H(t) = \frac{i}{\hbar} [\hat{H}, \vec{A}^H(t)]$	$\frac{d}{dt} \vec{A}^H(t) = \vec{A}^H(t) \Omega$	

$$\vec{A}^H(t) = \vec{A}^T e^{\Omega t} \quad \text{row vector}$$

$$\frac{d}{dt} \vec{A}^H(t) = \vec{A}^T \frac{d}{dt} e^{\Omega t} = \underbrace{\vec{A}^T e^{\Omega t}}_{\vec{A}^H(t)} \Omega$$

adjoint ("Heis") equation:  $\frac{d}{dt} \vec{A}^H(t) = \vec{A}^H(t) \Omega$

payoff: write adjoint eqn. in comp. form:

$$\frac{d}{dt} \vec{A}_n^H(t) = \sum_m A_m^H(t) \Omega_{mn}$$

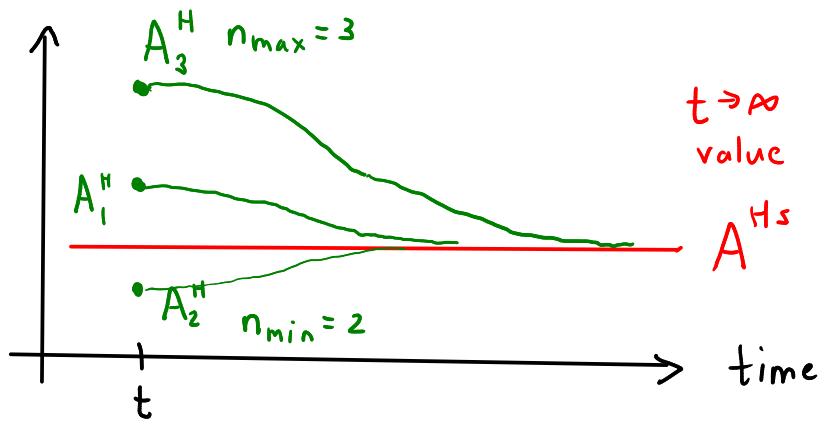
$$\Omega_{nn} = - \sum_{m \neq n} \Omega_{mn}$$

$$= \sum_{m \neq n} A_m^H(t) \Omega_{mn} + A_n^H(t) \left( - \sum_{m \neq n} \Omega_{mn} \right)$$

$$\frac{d}{dt} A^H(t) = \sum_{m \neq n} (A_m^H(t) - A_n^H(t)) \Omega_{mn}$$

Consider time  $t$   
Where values are

$$A_n^H(t)$$



at any time  $t$ ,  $n = n_{\max}$  has largest  $A_n^H(t)$   
 $n = n_{\min}$  has smallest  $A_n^H(t)$

$$\frac{d}{dt} A_{n_{\max}}^H = \sum_{m \neq n_{\max}} \underbrace{(A_m^H - A_{n_{\max}}^H)}_{< 0} \underbrace{\Omega_{mn_{\max}}}_{\text{at least one } m \text{ exists where } \Omega_{mn_{\max}} > 0}$$

$$< 0 \quad \begin{matrix} \text{(top curve} \\ \text{decreases)} \end{matrix} \quad \begin{matrix} \Omega_{mn_{\max}} > 0 \\ \text{b/c it's a} \\ \text{connected graph} \end{matrix}$$

$$\frac{d}{dt} A_{n_{\min}}^H = \sum_{m \neq n_{\min}} \underbrace{(A_m^H - A_{n_{\min}}^H)}_{> 0} \underbrace{\Omega_{mn_{\min}}}_{\text{at least one } m \text{ where } \Omega_{mn_{\min}} > 0}$$

$$> 0 \quad \begin{matrix} \text{(bottom curve} \\ \text{increases)} \end{matrix}$$

$$\Rightarrow A_n^H(t) \xrightarrow{t \rightarrow \infty} A^{Hs} \quad \text{for all } n$$