

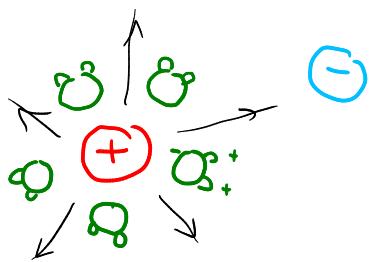
Pair of +e and -e ions:

$$\text{Coulomb energy} = \frac{-e^2}{4\pi\epsilon_0 r} \rightarrow \text{distance}$$

ϵ = permittivity of surrounding material

$$= \epsilon_r \epsilon_0$$

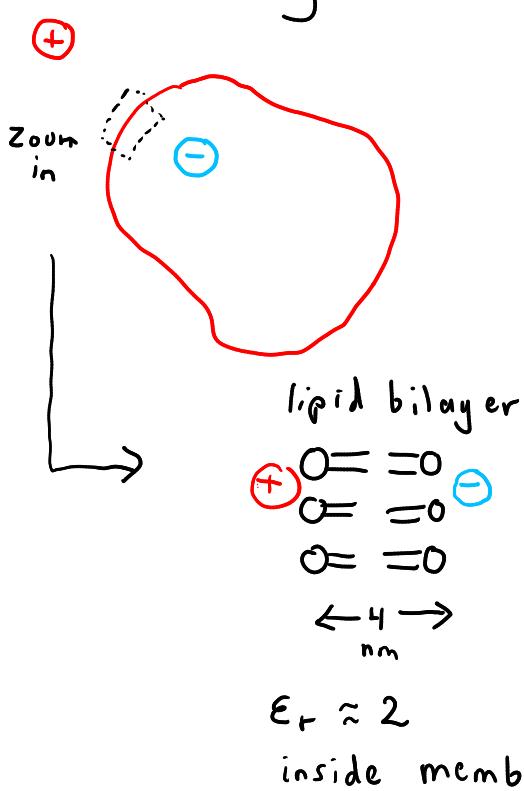
↓ vacuum perm.
 ↓ relative perm. $\left\{ \begin{array}{l} \approx 1 \text{ air} \\ \approx 80 \text{ water} \end{array} \right.$



water molec. reorient to weaken overall field seen by \ominus charge

$$\text{typical } r = 1 \text{ nm}, \epsilon_r = 80 \Rightarrow \text{energy} \sim -0.7 k_B T$$

\Rightarrow weak attraction easily broken by thermal fluctuations



imagine an extra pos. on outside + extra neg. charge on inside

Coulomb energy

$$\sim \frac{-e^2}{4\pi\epsilon_r\epsilon_0 r} = -0.2 \text{ eV}$$

$\uparrow \quad \uparrow$
 $2 \quad 8.8 \times 10^{-12} \text{ F/m}$

quite strong, stable against thermal fluctuations

Many extra charges \Rightarrow build up on either side of membrane

\Rightarrow locally looks like a parallel plate capacitor

charge density

$$\rho = \frac{Q}{A} \quad \text{charge area}$$

$$\text{voltage drop } V = \frac{\rho d}{\epsilon}$$

d = plate separation $\sim 4 \text{ nm}$

ϵ = permittivity $\sim 2\epsilon_0$

capacitance $C = \frac{Q}{V} = \frac{\epsilon A}{d}$

specific capacitance
(per unit area) $\frac{C}{A} = \frac{\epsilon}{d} \sim 1 \mu\text{F/cm}^2$

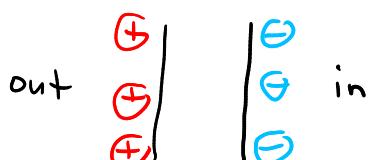
typical voltages across cell membranes:

$$V \approx -50 \text{ to } -200 \text{ mV}$$

measure

from outside to in

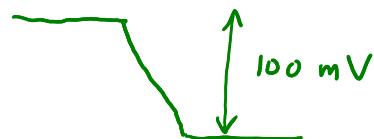
$$V \approx -100 \text{ mV}$$



typical area $A \sim 3 \mu\text{m}^2$

potential:

$$\Rightarrow \frac{Q}{e} = \frac{CV}{e} = 2 \times 10^4 \text{ charges on surface}$$

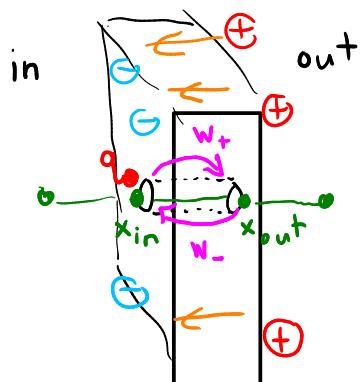


total ion conc. in cell $\sim 100 \text{ mM}$

$\approx 10^8 \text{ charges / fL}$

(bact. cell \approx fL)

energy cost of transporting charges:



LDB:

$$\frac{W_+}{W_-} = e^{-\beta(E_{out} - E_{in} + k_B T \ln \frac{C_{out}}{C_{in}})}$$

$$= e^{-\beta(-q \Delta V + k_B T \ln \frac{C_{out}}{C_{in}})}$$

$$\Delta V = V_{in} - V_{out} = -100 \text{ mV}$$

$$\text{scale } |e \Delta V| \approx 4 k_B T$$

note: when term in $() = 0 \Rightarrow W_+ = W_-$
no net charge current

$$-q \Delta V + k_B T \ln \frac{C_{out}}{C_{in}} = 0$$

$$\Delta V = \frac{k_B T}{q} \ln \frac{C_{out}}{C_{in}} \equiv V_N \quad \text{Nernst potential}$$

specific to each type of charge

rewrite LDB: $\frac{W_+}{W_-} = e^{-\beta q(V_N - \Delta V)}$

if $\Delta V > V_N \Rightarrow W_+ > W_-$ net flow out

$\Delta V < V_N \Rightarrow W_+ < W_-$ net flow in

imagine starting $\Delta V > V_N$ $q = +e$

\Rightarrow net flow out leads more excess \oplus outside

\Rightarrow decrease in ΔV ($C_{in} + C_{out} \approx \text{same}$
 $\Rightarrow V_N \approx \text{same}$)

\Rightarrow eventually ΔV reaches V_N
+ flow stops

opposite argument also works: $\Delta V < V_N$

$\Rightarrow \Delta V$ increases
until flow stops