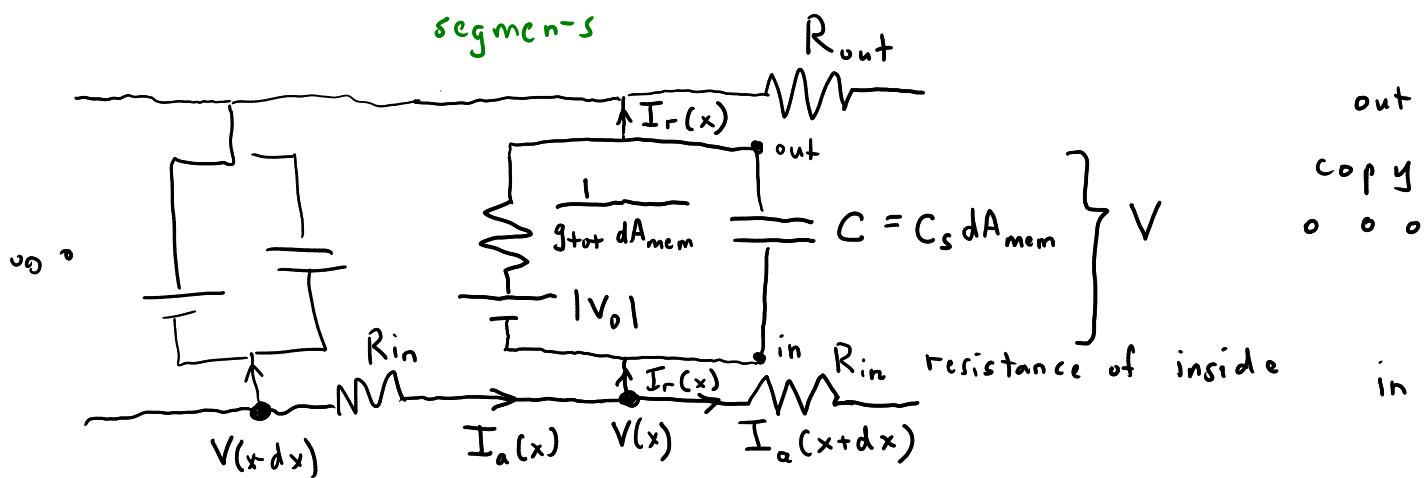




segments



$$R_{in} = \frac{dx}{\kappa \pi p^2} \quad R_{out} \approx 0$$

Potential change over one segment

$$dV = -I_a(x) R_{in} = -\frac{dx}{\kappa \pi p^2}$$

$$\Rightarrow I_a(x) = -\kappa \pi p^2 \frac{dV}{dx} \quad (1)$$

Current conservation: $\underbrace{I_a(x+dx) - I_a(x)}_{\approx -dx \frac{dI_a}{dx}} = I_r(x)$

$$\Rightarrow -dx \frac{dI_a}{dx} = \underbrace{dA_{mem} g_{tot} (V - V_0)}_{2\pi p dx} + dA_{mem} C_s \frac{dV}{dt}$$

$$\Rightarrow \frac{dI_a}{dx} = -2\pi p \left[g_{tot} (V - V_0) + C_s \frac{dV}{dt} \right] \quad (2)$$

Plug (1) into (2):

$\dot{I}_{tot}(V) = \text{total ion current thru membrane}$

$$\kappa \pi p^2 \frac{d^2V}{dx^2} = 2\pi p \left[\underbrace{g_{tot} (V - V_0)}_{\dot{I}_{tot}(V)} + C_s \frac{dV}{dt} \right]$$

simplify: $v(x, t) = V(x, t) - V_0 \quad \lambda = \sqrt{\frac{\rho K}{2g_{tot}}} \quad [\text{length}]$

$$\Rightarrow \lambda^2 \frac{\partial^2 u}{\partial x^2} - \tau \frac{\partial u}{\partial t} = u$$

$$\tau = \frac{C_s}{g_{tot}} \text{ [time]}$$

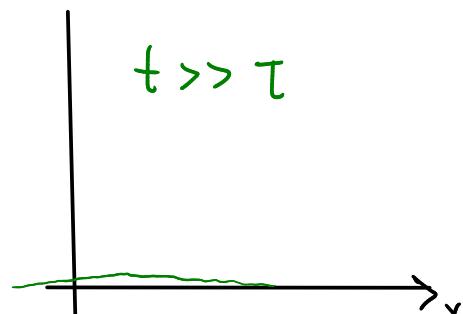
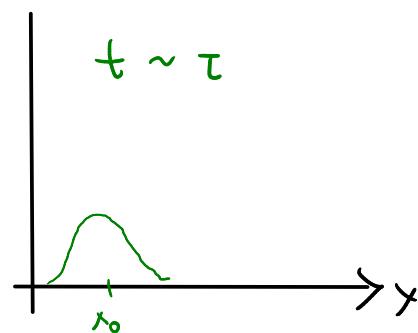
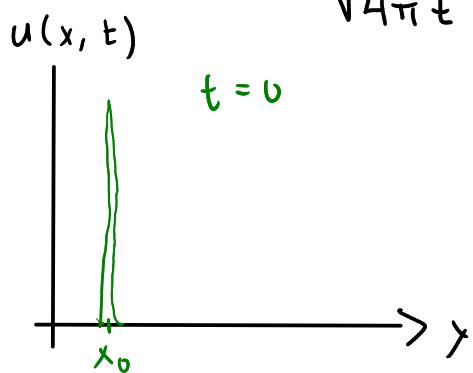
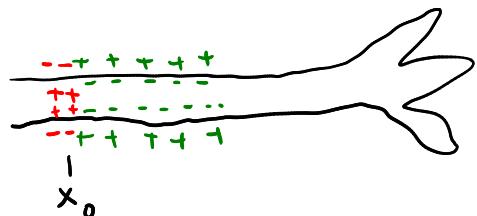
cable
equation

trivial sol'n: $u(x, t) = 0$ (everything is at rest)

more complicated sol'n: $V(x, t) = V_0$ everywhere

$$u(x, t=0) = B \delta(x-x_0)$$

$$u(x, t) = \frac{e^{-t/\tau}}{\sqrt{4\pi t \lambda^2 \tau^{-1}}} \exp \left[-\frac{(x-x_0)^2}{4 + \lambda^2 \tau^{-1}} \right]$$



speed of spread: $\frac{\text{dist}}{\text{time}} \sim \frac{\lambda}{\tau} = 9 \text{ m/s}$

squid:

$$\rho = 0.5 \text{ mm}$$

$$K = 3 \Omega^{-1} \text{ m}^{-1}$$

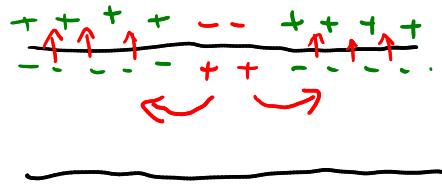
$$g_{tot} = 10 \Omega^{-1} \text{ m}^{-2}$$

$$C_s = 1 \mu\text{F/cm}^2$$

$$\Rightarrow \lambda = 9 \text{ mm}$$

$$\tau = 1 \text{ ms}$$

Reason for decay:



excess charge is spreading

but also leaking / pumped out

thru membrane

Solution: make membrane channels responsive to voltage

$$j_{tot}(V) = g_{tot} (V - V_o)$$

we derived this assuming most Na channels are closed : $g_{Na} \ll g_K$

$$V_o = \frac{2g_{Na}V_N^{Na} + 3g_KV_N^K}{2g_{Na} + 3g_K}$$

$$V_N^K = -75 \text{ mV}$$

$$V_N^{Na} = 54 \text{ mV}$$

$$\approx -65 \text{ mV} \quad (\text{close to } V_N^K)$$

if Na channels mostly open:

$$\begin{matrix} \text{new} \\ \text{value} \end{matrix} \quad \tilde{g}_{Na} >> g_K \quad (\text{many more} \\ \text{Na channels})$$

$$\Rightarrow \tilde{V}_o = 40 \text{ mV} \quad (\text{close to } V_N^{Na})$$

