	quantum	master equ
description of state	$\|\psi(t)\rangle$ vector in Hilbert space	$\vec{p}(t)$ vector in prob. space
dynamical equation	$i\hbar \frac{\partial}{\partial t}\|\psi(t)\rangle = \hat{H}\|\psi(t)\rangle$	$\frac{\partial}{\partial t}\vec{p}(t) = \Omega \vec{p}(t)$
solution	$\|\psi(t)\rangle = \underbrace{e^{i\hat{H}t/\hbar}}_{\text{propogator } \hat{U}(t)}\|\psi(0)\rangle$	$\vec{p}(t) = e^{\Omega t}\vec{p}(0)$
avg. of observable	$A(t) = \langle\psi(t)\|\hat{A}\|\psi(t)\rangle$ $= \langle\psi(0)\|\hat{A}^H(t)\|\psi(0)\rangle$	$A(t) = \vec{A}\cdot\vec{p}(t)$ $= \vec{A}^H(t)\cdot\vec{p}(0)$
Heis. dynamical equation	$\frac{d}{dt}\vec{A}^H(t) = \frac{i}{\hbar}[\hat{H}, \vec{A}^H(t)]$	$\frac{d}{dt}\vec{A}^H(t) = \vec{A}^H(t)\Omega$

observables in classical systems: state var A_n

vector $\vec{A} \Rightarrow$ comp's A_n

$$A(t) = \sum_n A_n p_n(t)$$
$$= \vec{A}\cdot\vec{p}(t)$$

recall Heisenberg picture:

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle$$
$$\langle\psi(t)| = \langle\psi(0)|\hat{U}^\dagger(t)$$

$$A(t) = \langle\psi(t)|\hat{A}|\psi(t)\rangle$$
$$= \langle\psi(0)|\hat{U}^\dagger(t)\hat{A}\hat{U}(t)|\psi(0)\rangle$$

$$\equiv \hat{A}^H(t) \quad \text{Heisenberg oper.}$$

"classical" Heisenberg: $\quad A(t) = \sum_n A_n\, p_n(t)$

$$\vec{p}(t) = e^{\Omega t}\,\vec{p}(0) \qquad\qquad = \sum_n A_n \left(e^{\Omega t}\,\vec{p}(0)\right)_n$$

$$p_n(t) = \left(e^{\Omega t}\,\vec{p}(0)\right)_n \qquad\qquad = \sum_{n,m} A_n \left(e^{\Omega t}\right)_{nm} p_m(0)$$

$$A^H_m(t) \equiv \sum_n A_n \left(e^{\Omega t}\right)_{nm} \qquad\qquad \equiv \sum_m A^H_m(t)\, p_m(0)$$

$$\vec{A}^H(t) = \vec{A}^T e^{\Omega t} \qquad\qquad = \vec{A}^H(t)\cdot\vec{p}(0)$$

$$\frac{d}{dt}\vec{A}^H(t) = \vec{A}^T \frac{d}{dt}\underbrace{e^{\Omega t}}_{\mathbb{I}+\Omega t+\frac{\Omega t^2}{2!}+\cdots} = \underbrace{\vec{A}^T e^{\Omega t}}_{\vec{A}^H(t)}\,\Omega$$

"Heis." dynamical eqn (adjoint master equ.): $\quad \frac{d}{dt}\vec{A}^H(t) = \vec{A}^H(t)\,\Omega$

---

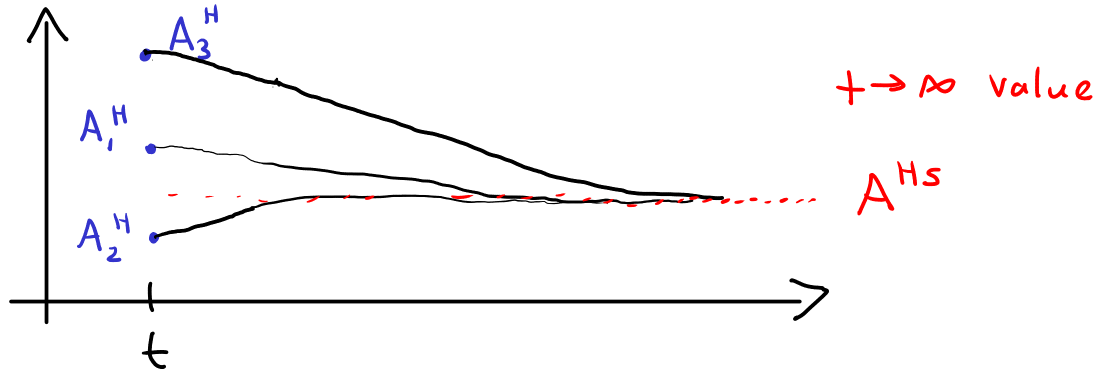
payoff: write adjoint eqn in comp. form

$$\Omega_{nn} = -\sum_{m\neq n}\Omega_{mn}$$

$$\frac{d}{dt}A^H_n(t) = \sum_m A^H_m(t)\,\Omega_{mn}$$

$$= \sum_{m\neq n} A^H_m(t)\,\Omega_{mn} + A^H_n(t)\left(-\sum_{m\neq n}\Omega_{mn}\right)$$

$$\frac{d}{dt}A^H_n(t) = \sum_{m\neq n}\left(A^H_m(t) - A^H_n(t)\right)\Omega_{mn}$$



at time t: $n = n_{max}$ has largest $A_n^H(t)$
$n = n_{min}$ has smallest $A_n^H(t)$

$$\frac{d}{dt} A^H_{n_{max}} = \sum_{m \neq n_{max}} \underbrace{(A_m^H - A^H_{n_{max}})}_{<0} \underbrace{\Omega_{m n_{max}}}_{\substack{\text{at least one} \\ m \text{ where} \\ \Omega_{m n_{max}} > 0 \\ \text{(b/c connected graph)}}}$$

< 0

top curve always decreases

$$\frac{d}{dt} A^H_{n_{min}} = \sum_{m \neq n_{min}} \underbrace{(A_m^H - A^H_{n_{min}})}_{>0} \underbrace{\Omega_{m n_{min}}}_{\substack{\text{at least one } m \\ \text{where } > 0}}$$

> 0

bottom curve increases

$$\Rightarrow \quad A_n^H(t) \xrightarrow[t \to \infty]{} A^{Hs} \quad \text{for any } n$$

arguments for any $\vec{A}^H(t)$

create "indicator" observable: $\vec{A}^{(k)} \Rightarrow A_n^{(k)} = \delta_{nk} = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$

avg: $A^{(k)}(t) = \sum_n A_n^{(k)} p_n(t) = p_k(t)$

$$A_n^{(k)H} \xrightarrow[t\to\infty]{} A^{(k)Hs} \quad \text{const.}$$

$$\Rightarrow \boxed{p_k(t) \xrightarrow[t\to\infty]{} p_k^s \quad \text{limiting value}}$$

for const. Ω + connected graph

$\Rightarrow$ sys. will go to stationary state

implications:

$$F(t) = E(t) - T\,S(t) \qquad \dot{F}(t) = \frac{d}{dt}F(t)$$

combined 1st + 2nd law: $\dot{F}(t) = \dot{W}(t) - T\,\dot{I}(t)$

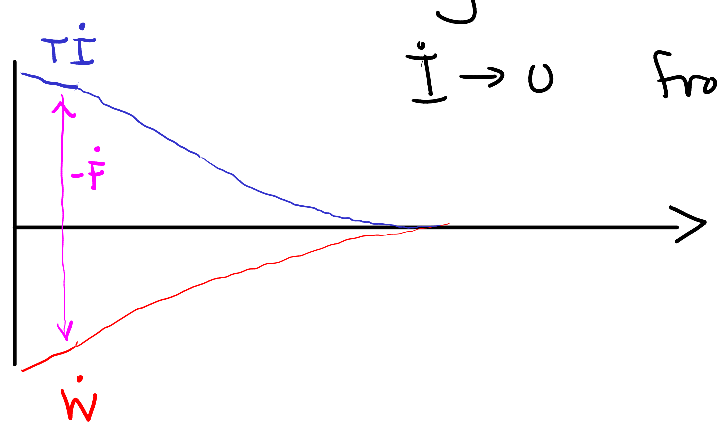
$t \to \infty$: stationary state $p_n(t) \to p_n^s$

$$\dot{E}, \dot{S}, \dot{F} \to 0$$

$$\Rightarrow \dot{W} - T\dot{I} \to 0$$

case I: $\dot{W} < 0$ sys. does net work on outside

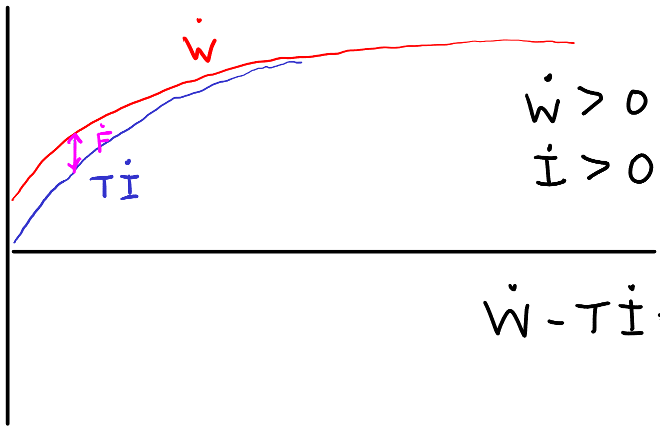
$\dot{I} > 0$ by construction



$\dot{I} \to 0$ from above $\Rightarrow \dot{F} \to 0$

time $\Rightarrow$ ESS

case II: $\dot{W} > 0$ as $t \to \infty$ (continuous supply of work from outside, so that net work > 0 on sys)



$$\dot{W} - T\dot{I} \to 0 \Rightarrow \dot{W} = T\dot{I} > 0$$

NESS

$$\underset{\substack{\text{net}\\\text{power}\\\text{int. sys}}}{\dot{W}} = \underset{\substack{\text{input}\\\text{power}}}{P_{in}} - \underset{\substack{\text{output}\\\text{power}}}{P_{out}} = \underbrace{T\dot{I}}_{P_{diss}} > 0 \quad t \to \infty$$

(all > 0 terms) (all < 0 terms)

dissipated (lost) power

any interesting biology: $P_{diss} > 0$

$\dot{I} > 0$ NESS