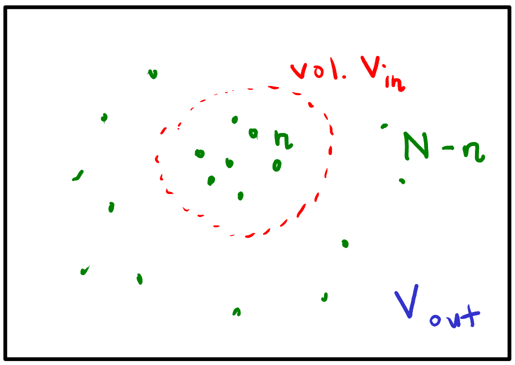
Transport across membranes

I. Uncharged molecules



vol. $V_{tot} = V_{in} + V_{out}$

for simplicity: V_{out}, V_{in} fixed
N fixed

model: "state" n

$\Rightarrow$ n molec. inside
$N-n$ molec. outside

$$\cdots \quad (n-1) \underset{k_{out}(n)}{\overset{k_{in}(n-1)}{\rightleftharpoons}} (n) \underset{k_{out}(n+1)}{\overset{k_{in}(n)}{\rightleftharpoons}} (n+1) \quad \cdots$$

no enthalpy diff. b/t states

LDB: $$\frac{k_{out}(n+1)}{k_{in}(n)} = e^{-\beta\left(\mu_{out}(n+1) - \mu_{in}(n)\right)}$$

$$\mu_{out}(n) = \mu_0 + k_B T \ln \overbrace{c_{out}(n)}^{\frac{N-n}{V_{out}}}$$

$$\mu_{in}(n) = \mu_0 + k_B T \ln \underbrace{c_{in}(n)}_{n/V_{in}}$$

$$\frac{k_{out}(n+1)}{k_{in}(n)} = \frac{c_{in}(n)}{c_{out}(n+1)}$$

$n \gg 1$

$k_{out}(n+1) \approx k_{out}(n)$

etc.

$$\Rightarrow \quad \frac{k_{out}(n)}{k_{in}(n)} \approx \frac{c_{in}(n)}{c_{out}(n)}$$

net rate of molec. leaving cell (V_{in}):

$$F(n) = k_{out}(n) - k_{in}(n)$$
$$= k_{out}(n)\left(1 - \frac{k_{in}(n)}{k_{out}(n)}\right)$$
$$\approx k_{out}(n)\left(1 - \frac{c_{out}(n)}{c_{in}(n)}\right)$$
$$= \frac{k_{out}(n)}{c_{in}(n)}\left(c_{in}(n) - c_{out}(n)\right)$$

equil. state n_0 where $c_{in}(n_0) = c_{out}(n_0)$

$$\Rightarrow \quad \frac{n_0}{V_{in}} = \frac{N - n_0}{V_{out}}$$

$F = 0$

$$n_0 = N \frac{V_{in}}{V_{in} + V_{out}}$$
$$= N \frac{V_{in}}{V_{tot}}$$

Taylor expand $F(n)$ around $n = n_0$:

$$\frac{k_{out}(n)}{c_{in}(n)} \approx \frac{k_{out}(n_0)}{c_{in}(n_0)} + \cdots$$

$$k_0 = k_{out}(n_0)$$
$$c_0 = \frac{N}{V_{tot}}$$

$$\approx \frac{k_0}{c_0} + \cdots$$

$$c_{in}(n) - c_{out}(n) \approx O(n - n_0) + \cdots$$

$$F(n) \approx \frac{k_0}{c_0}\left(c_{in}(n) - c_{out}(n)\right)$$

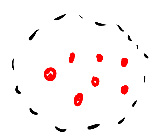
net flux of particles leaving

$$\frac{F(n)}{A} = \overbrace{\frac{k_0}{c_0 A}}^{p}\left(c_{in}(n) - c_{out}(n)\right)$$

area of membrane

$$p = \frac{k_0}{c_0 A} = \frac{s^{-1}}{nm^{-3}\ nm^{2}} = \frac{nm}{s}$$ permeability const.

rough estimates of how quickly leak out of cells:



$V_{out} \to \infty$

$c_{out} \to 0$

$n_0 \to 0$

dynamics of mean # of part. in cell

$$\frac{dn}{dt} \approx -F(n) = -pA\,c_{in}(n)$$

divide by V_{in}

$$\frac{dc_{in}}{dt} = -\frac{pA}{V_{in}}\,c_{in}$$

$$\Rightarrow \quad c_{in}(t) = c_{in}(0)\,e^{-t/\tau}$$

$$\tau = \frac{\overset{nm^3}{V_{in}}}{\underset{nm/s}{p}\ \underset{nm^2}{A}}$$ time const.

~ characteristic time of leaking out

two ends of spectrum:

Na^+ : $p \sim 10^{-5}$ nm/s

$R \sim 1000$ nm

$\Rightarrow \tau \sim 3 \times 10^{7}\ s > 1\ yr$

H_2O: $\quad p \sim 10^5$ nm/s ← essentially impermeable

$\Rightarrow \tau \sim 3 \times 10^{-3}$ s ← very permeable

takeaway: certain molecules "stuck", certain molecules pass easily