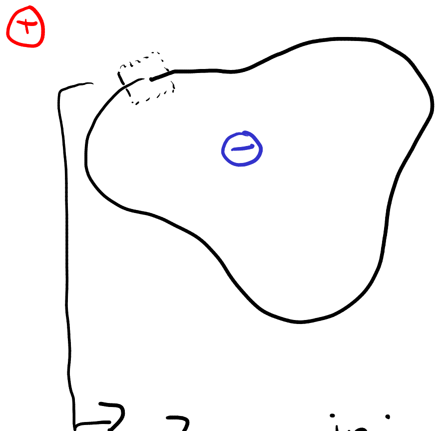
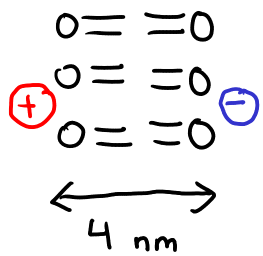


Zoom in:



$\varepsilon_r \approx 2$ inside membrane (vs. 80 for water)

Coulomb energy

$$\sim \frac{e^2}{4\pi\varepsilon_r\varepsilon_0 r} = -0.2 \text{ eV} = -8\ k_B T$$

Stable against thermal fluctuations

if you introduce excess charge on inside/outside (i.e. via pumping) $\Rightarrow$ eventually these charges get stuck in pairs across membrane

$\Rightarrow$ looks like parallel plate capacitor

$$\rho = \frac{Q}{A} = \frac{\text{charge}}{\text{area}} \qquad V = \frac{\rho d}{\varepsilon}$$

d = separation of plates

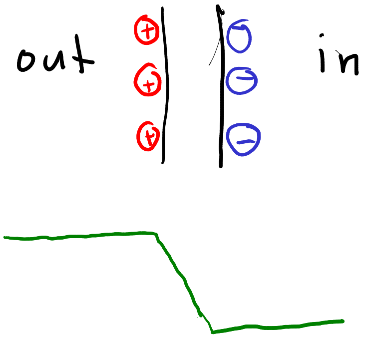
ε = permittivity $= \varepsilon_r \varepsilon_0$

$$C = \text{capacitance} = \frac{Q}{V} = \frac{\varepsilon A}{d}$$

Specific capacitance $\dfrac{C}{A} = \dfrac{\varepsilon}{d} \sim 1\ \mu F/cm^2$

typical voltage across membranes:

meas. from outside to in $\quad V \approx -50$ to -200 mV



let's say $V = -100$ mV

$A \sim 3\ \mu m^2$

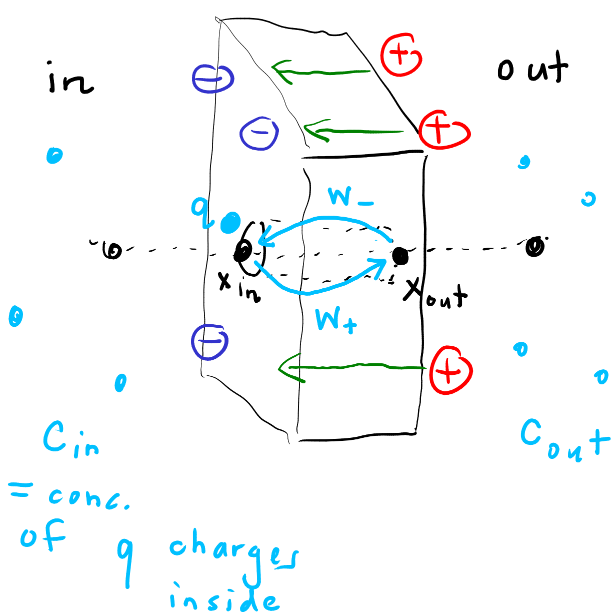
$$\frac{Q}{e} = \frac{CV}{e} = 2 \times 10^4 \text{ charges on surface}$$

total ion conc. $\sim$ 100 mM

$\approx 10^8$ charges per fL

bacterial cell $\approx$ fL

transport of ions through channels ("holes in membrane"):



det. balanced:

$$\frac{W_+}{W_-} = e^{-\beta\left(\underbrace{E_{out} - E_{in}}_{\text{energy diff. due to membrane field}} + \overbrace{k_BT \ln \frac{C_{out}}{C_{in}}}^{\text{chem. potential}}\right)}$$

$$= e^{-\beta\left(-q\Delta V + k_BT \ln \frac{C_{out}}{C_{in}}\right)}$$

$$\Delta V = V_{out} - V_{in} = -100 \text{ mV}$$

$$|e\Delta V| \approx 4\, k_BT$$

when term in $(\ ) = 0 \Rightarrow W_+ = W_-$

no net charge current

$$-q\Delta V + k_B T \ln \frac{C_{out}}{C_{in}} = 0$$

$$\Delta V = \frac{k_B T}{q} \ln \frac{C_{out}}{C_{in}} \equiv V_N$$

Nernst potential (depends on q)

rewrite det. balance:

$$\frac{W_+}{W_-} = e^{-\beta q (V_N - \Delta V)}$$

if $\Delta V > V_N \Rightarrow W_+ > W_-$ net flow out

$\Delta V < V_N \Rightarrow W_- < W_+$ net flow in

imagine we start $\Delta V > V_N$ $q = +e$

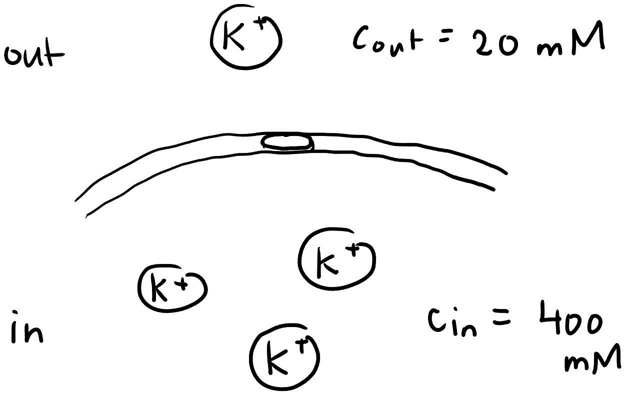
net flow out $\Rightarrow$ more excess pos. ions outside

ΔV becomes more negative (decreases)

C_{in} & $C_{out} \approx$ same $\Rightarrow V_N \approx$ same

eventually ΔV reaches $V_N \Rightarrow$ flow stops

Same argument for $\Delta V < V_N \Rightarrow \Delta V$ increase until flow stops



focus on one ion type: K^+ [charge $q = +e$]

$$\Rightarrow V_N = \frac{k_B T}{q} \ln \frac{c_{out}}{c_{in}} = -75 \text{ mV}$$

typical $\Delta V = -65$ mV

net current out of cell $I \propto W_+ - W_-$

Taylor expand: $I = G(\Delta V - V_N) + \dots$

membrane conductivity

$$\frac{\text{current}}{\text{area}} = j = \frac{I}{A} = g(\Delta V - V_N)$$

$$g = \frac{G}{A} \approx 10\ \Omega^{-1} m^{-2}$$

for K^+ channels in squid axons

reality: multiple ion types, each w/ own channels

for type i: $$j_i = g_i(\Delta V - \underbrace{V_N^{(i)}}_{\frac{k_B T}{q_i} \ln \frac{c_{out}^{(i)}}{c_{in}^{(i)}}})$$

function of # channels/area + whether channels open or closed two most important types: K^+, Na^+ $\quad q_i = +e$

	C_{in}	C_{out}	$V_N^{(i)}$
Na^+	50 mM	440 mM	54 mV
K^+	400 mM	20 mM	-75 mV

squid equil.

to get equilibrium (no current flow) need pumping of Na + K ions: $\Delta V = V_0 = -65$ mV

$$0 = j_{Na} = g_{Na}\underbrace{(V_0 - V_N^{Na})}_{<0} + \underset{>0}{j_{Na}^{pump}}$$

pumping current for Na ions

$$0 = j_K = g_K \underbrace{(V_0 - V_N^K)}_{>0} + \underset{<0}{j_K^{pump}}$$

$\Rightarrow$ need Na pumped out + K pumped in to balance out + reach equil.
(in additon to passive channels)