In order to achieve a resting potential

$V_0 = -65\ mV$ between $V_N^{Na} = +54\ mV$

and $V_N^K = -75\ mV$

$\Rightarrow$ we need pumping Na pumped out

+ K pumped in

Pump: Na/K pump takes 3 Na^+ out +
2 K^+ in for every cycle,
consuming one ATP

Neuron/kidney cells: 50-70% of their total ATP consumption is used by these pumps

equil:

$$0 = \dot{j}_{Na} = g_{Na}(V_0 - V_N^{Na}) + \dot{j}_{Na}^{pump}$$

$$0 = \dot{j}_K = g_K(V_0 - V_N^K) + \dot{j}_K^{pump}$$

$$\dot{j}_K^{pump} = -\frac{2}{3}\dot{j}_{Na}^{pump}$$

$\Rightarrow$ solve for $V_0 = \dfrac{2g_{Na}V_N^{Na} + 3g_K V_N^K}{2g_{Na} + 3g_K}$

plug in $V_N^{Na} = 54\ mV$, $V_N^K = -75\ mV$

normal resting state: $g_K >> g_{Na}$ (most Na channels closed)

$$g_K = 25\, g_{Na}$$

$\Rightarrow$ V_0 is closer to V_N^K

$V_0 \approx -65\,mV$

What happens if membrane potential $\Delta V \neq V_0$?

$$\Delta V = V_0 + \underset{\text{perturbation}}{u}$$

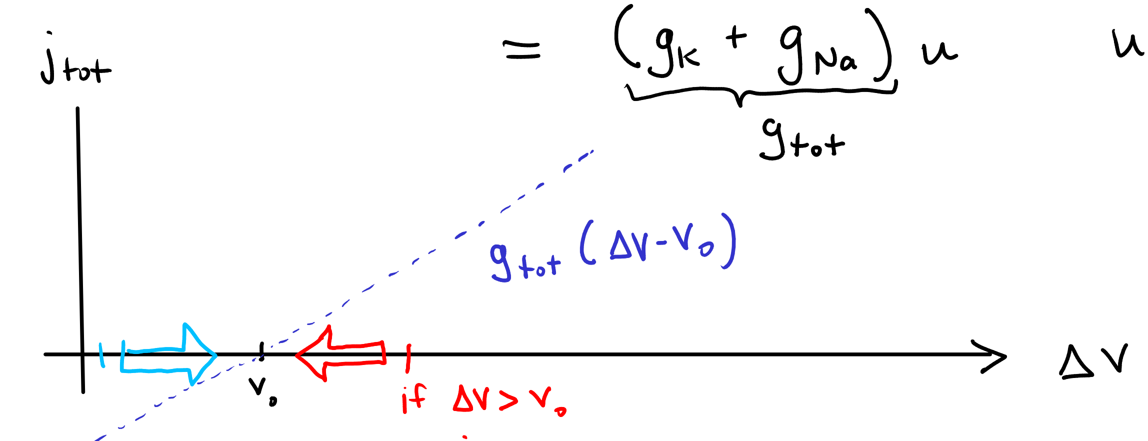
$u \neq 0$

$u > 0$ (put more - charges outside relative to resting state)

$$j_{tot} = j_K + j_{Na} = g_K(u + V_0 - V_N^K) + j_K^{pump} + g_{Na}(u + V_0 - V_N^{Na}) + j_{Na}^{pump}$$

$$= \underbrace{(g_K + g_{Na})}_{g_{tot}} u$$

$u = \Delta V - V_0$



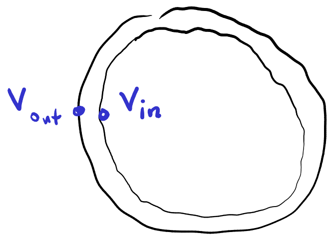
if $\Delta V < V_0$
$j_{tot} < 0$
ΔV bec. more positive

if $\Delta V > V_0$
$j_{tot} > 0$
ΔV becomes more negative

takeaway: any small perturbation gets returned to zero

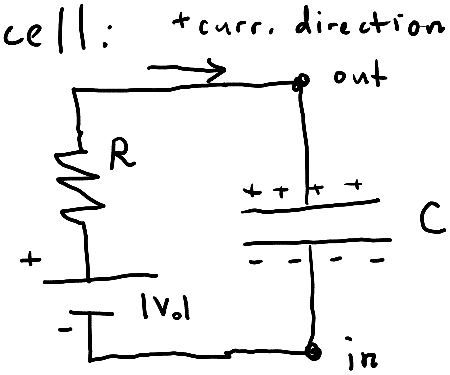
$\Rightarrow$ equilibrate

Build a circuit model for cell:



A_{mem} = memb. area

$V_0 = -65\,mV$
< 0



V
$= V_{in} - V_{out}$
$= V_0 + u$

$V = V_{in} - V_{out}$

$C_s = \frac{\text{capac.}}{\text{area}}$ $\quad C = C_s A_{mem}$ $\quad R = \frac{1}{g_{tot} A_{mem}}$

$$i = j_{tot} A_{mem} = \underbrace{g_{tot} A_{mem}}_{R^{-1}} (V - V_0)$$

minus sign due to opposite potential convention

$$= -C \frac{dV}{dt}$$

$$\Rightarrow \quad \frac{dV}{dt} = -\frac{1}{RC}(V - V_0)$$

solution: $V(t) = V_0 + e^{-t/RC}(V_{init} - V_0)$

$\uparrow$ initial voltage

$\Rightarrow \quad V(t) \to V_0$ as $t \gg RC$

$$RC = \frac{C_s A_{mem}}{g_{tot} A_{mem}} = \frac{C_s}{g_{tot}} \sim 1\ ms$$

$C_s \sim 1\,\mu F/cm^2 \qquad g_{tot} \sim 10\ \Omega^{-1} m^{-2}$

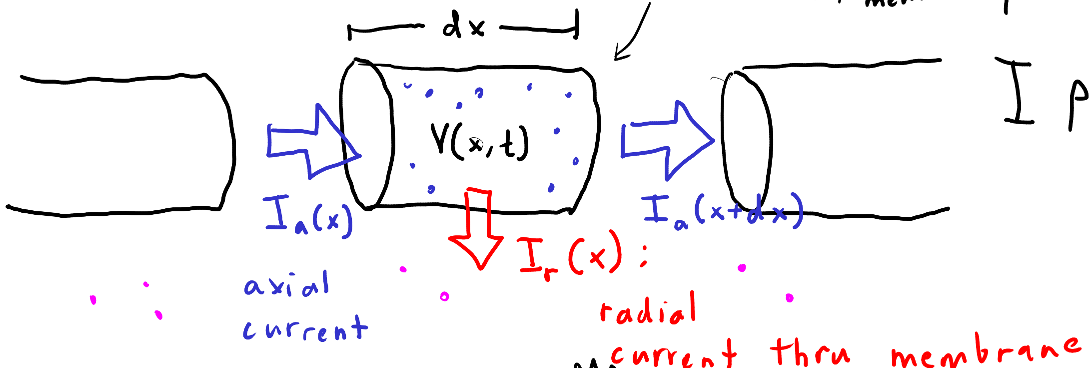
here we assumed cell is small so $V(t)$

describes uniform potential across whole cell

$\Rightarrow$ not true for neurons

how do we model really long axons for neurons?

area: $dA_{mem} = \pi \rho^2 \, dx$



$I \; \rho$