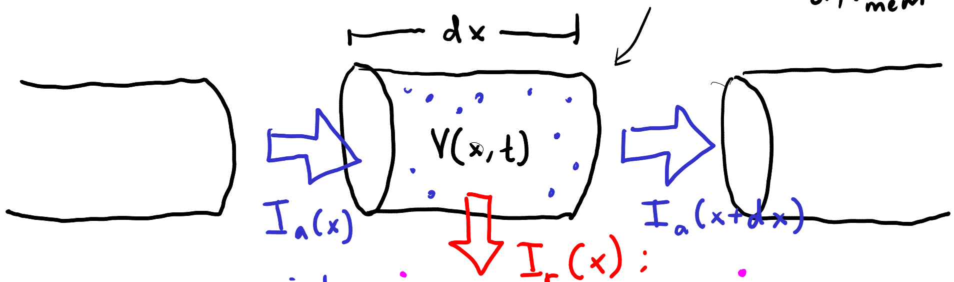
how do we model really long axons for neurons?

area: $dA_{mem} = \pi \rho^2 \, dx$

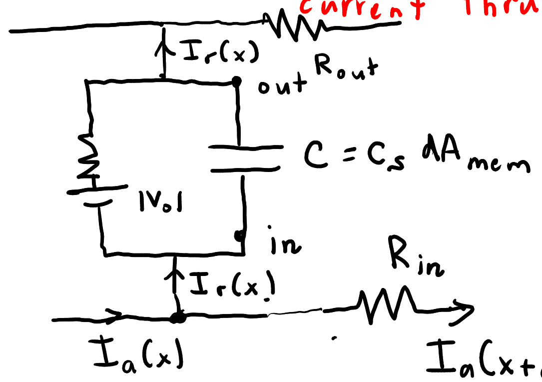


$I \quad \rho$

axial current

radial current thru membrane

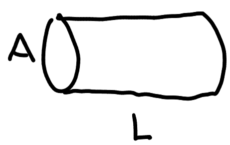
repeat . .



. . repeat . .

Pouillet's law: resistance of cylindrical wire

$$R = \frac{L}{\sigma A}$$



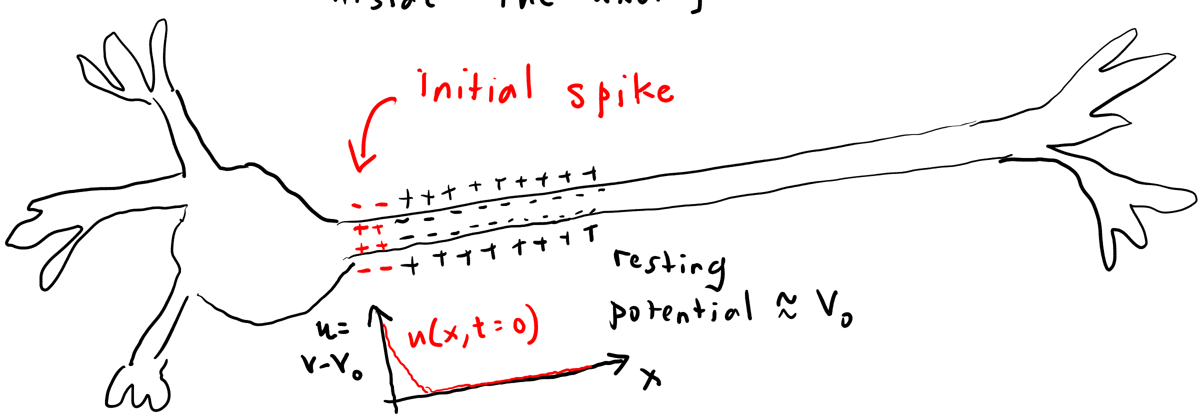
σ = conductivity (material property)

$$R_{in} = \frac{dx}{K \pi \rho^2}$$

K = conductivity of inside of axon $\propto$ conc. of total ions $\sim 3 \, \Omega^{-1} m^{-1}$

$$R_{out} \approx 0$$

(A is huge for outside the axon)



Question: does the spike propagate?
how fast?

potential change inside axon from one dx segment to next:

$$dV = -I_a(x)\, R_{in}$$

$$= -I_a(x) \frac{dx}{K\pi\rho^2}$$

$$\Rightarrow I_a(x) = -K\pi\rho^2 \frac{dV}{dx} \quad (1)$$

change in axial current from segm. to segm.

$$\underbrace{I_a(x) - I_a(x+dx)}_{\approx\, -dx \frac{dI_a}{dx}} = I_r(x)$$

$$= dA_{mem}\, g_{tot}(V - V_0) + dA_{mem}\, C_s \frac{dV}{dt}$$

$$\Rightarrow \quad -dx \frac{dI_a}{dx} = 2\pi\rho\, dx \left[g_{tot}(V - V_0) + C_s \frac{dV}{dt} \right] \quad (2)$$

plug (1) into (2), divide by dx:

$$K\pi\rho^2 \frac{d^2V}{dx^2} = 2\pi\rho \left[\underbrace{g_{tot}(V - V_0)}_{\substack{j_{tot}(V) = \\ \text{total ion current} \\ \text{thru memb. channels}}} + C_s \frac{dV}{dt} \right]$$

$$u(x,t) = V(x,t) - V_0 \qquad \lambda \equiv \sqrt{\frac{\rho K}{2 g_{tot}}} \quad \text{[units: length]}$$

$$\tau = \frac{C_s}{g_{tot}} \quad \text{[units: time]}$$

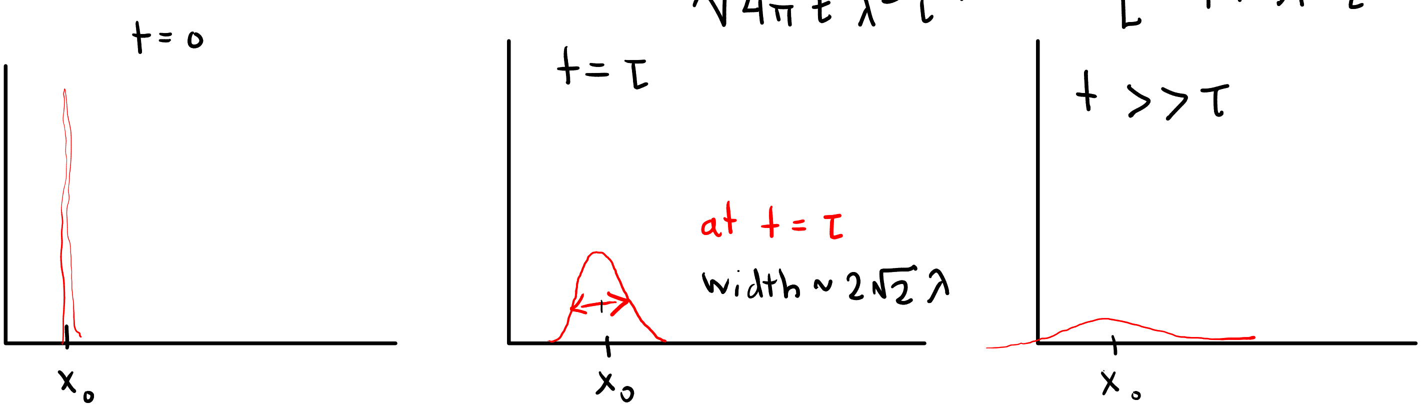
$$\Rightarrow \boxed{\lambda^2 \frac{\partial^2 u}{\partial x^2} - \tau \frac{\partial u}{\partial t} = u}$$

cable equation

trivial sol'n: $u(x,t) = 0$ everything at resting potential

initial spike: $u(x,0) = B\,\delta(x - x_0)$

solution: $$u(x,t) = \frac{e^{-t/\tau}}{\sqrt{4\pi t \lambda^2 \tau^{-1}}} \exp\left[-\frac{(x-x_0)^2}{4t\lambda^2\tau^{-1}} \right]$$



speed of spread $\frac{\text{dist}}{\text{time}} \sim \frac{\lambda}{\tau} \sim 9$ m/s

for squid: $\rho = 0.5$ mm $\qquad \lambda = 9$ mm

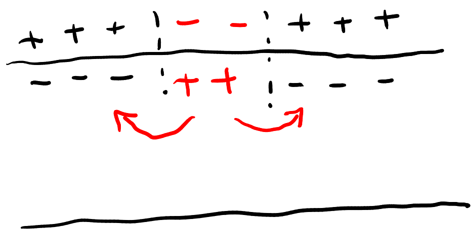
$K = 3\ \Omega^{-1} m^{-1}$ $\qquad \tau = 1$ ms

$g_{tot} = 10\ \Omega^{-1} m^{-2}$

$C_s = 1\ \mu F/cm^2$

Problem: potential decays!

Source:



excess charge spreads, then leaks / pumped out

$\Rightarrow$ potential returns to rest

$u(x, t\to\infty) \to 0$

Revisit: $j_{tot}(V) = g_{tot}(V - V_0)$ derived under assumptions most Na channels were closed

$$V_0 = \frac{2 g_{Na} \overset{54\,mV}{V_N^{Na}} + 3 g_K \overset{-75\,mV}{V_N^K}}{2 g_{Na} + 3 g_K}$$

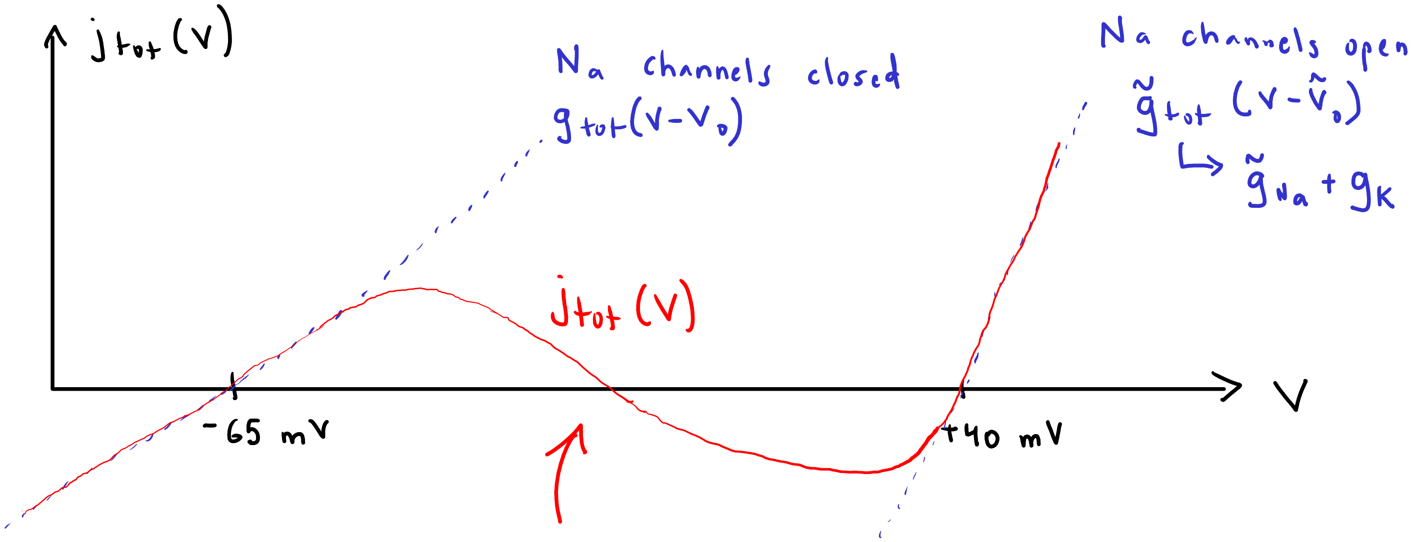
Na channels closed: $g_{Na} \ll g_K$

$V_0 = -65\ mV$

if Na channels mostly open:

new value $\tilde{g}_{Na} \gg g_K$ (many more Na channels)

$\Rightarrow \tilde{V}_0 = 40\ mV$ close to V_N^{Na}



this crossover occurs because $p_{Na}(V)$ = prob. of Na channels open depends on V

next time: cable eqn. w/ this nonlinear $j_{tot}(V)$