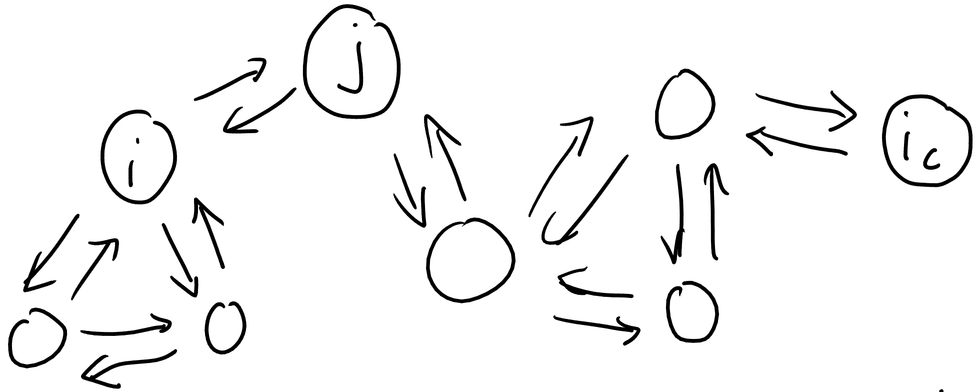
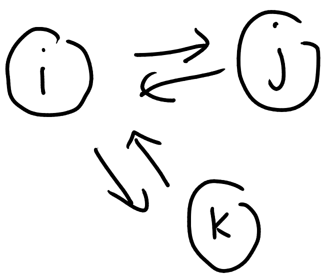
RECAP:




$\tau_i$ = avg. time it takes to reach $i_c$ for first time, given start at $i$

3 parts to solution:

1)




how long before we leave $i$ on average?

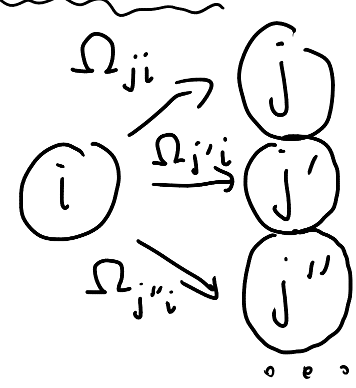
$\equiv \tau_i^{esc}$ escape time from $i$

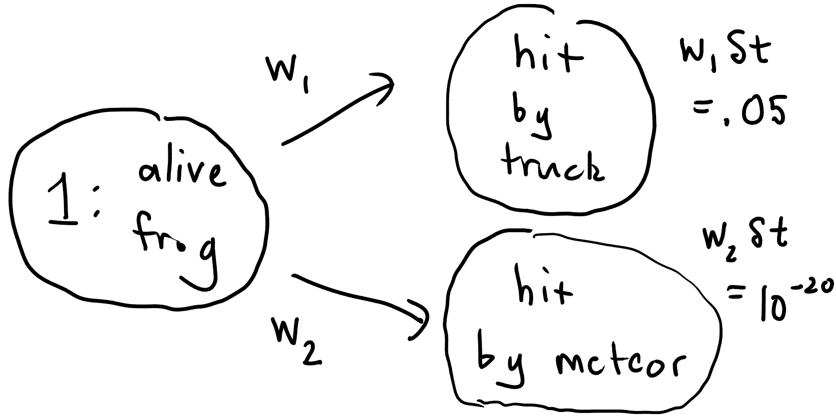
2) find prob. that we escaped to neighbor $j$ after leaving $i$ $\equiv \pi_{ji}$

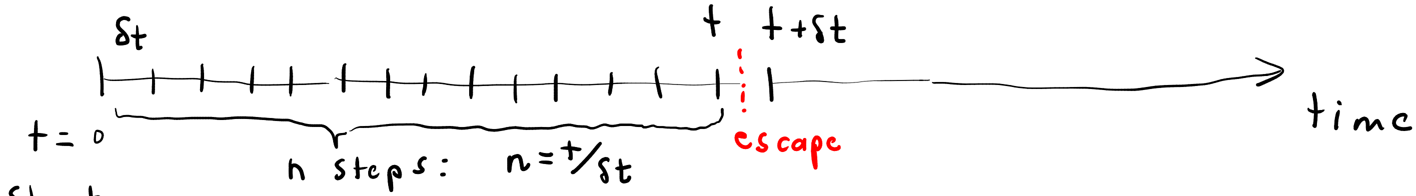
3) construct a recursive argument to find $\tau_i$ from 1) and 2)

$\delta t = 1s$

1)






start in state $i$

prob. of <u>not</u> leaving state $i$ in time step $\delta t$ $= 1 - (w_1 + w_2)\,\delta t$

in general $= 1 - \sum_{j \neq i} \Omega_{ji}\,\delta t$

$= 1 - |\Omega_{ii}|\,\delta t$

$$\Omega_{ii} = -\sum_{j\neq i} \Omega_{ji}$$

b/c columns of $\Omega$ sum to zero

$f_i(t)\,\delta t$ = prob. that you escape $i$ b/t time $t$ + $t+\delta t$

$$= \underbrace{\left[1 - |\Omega_{ii}|\,\delta t\right]^n}_{\text{prob. to survive } n \text{ steps}} \; \underbrace{|\Omega_{ii}|\,\delta t}_{\text{prob. to escape in last step}}$$

$\delta t = \dfrac{t}{n}$

$n \to \infty$, $\delta t \to$ small

$$\approx \exp\left(-|\Omega_{ii}|\,t\right) |\Omega_{ii}|\,\delta t$$

$$\Rightarrow \quad f_i(t) = e^{-|\Omega_{ii}|t}\,|\Omega_{ii}|$$

$$\int_0^\infty dt\; f_i(t) = \underline{1}$$

$T_i^{esc}$ = avg. time to escape $i$

$$= \int_0^\infty dt\; t\, f_i(t) = \frac{1}{|\Omega_{ii}|}$$

$$\text{frog:} \qquad \Omega = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \\ 2 & \\ 3 & \end{array}\!\!\!\!\!\!\!\!\begin{pmatrix} -w_1-w_2 & 0 & 0 \\ w_1 & 0 & 0 \\ w_2 & 0 & 0 \end{pmatrix}$$

$$\tau_1^{esc} = \frac{1}{|\Omega_{11}|} = \frac{1}{w_1+w_2} \approx \frac{1}{.05} = 20\text{ s}$$

2) After escaping, which state are we in?

frog: prob. of dying by truck (escaping to 2) $= \dfrac{w_1}{w_1+w_2} \approx 1 - 10^{-20}$

" " by meteor (escape to 3) $= \dfrac{w_2}{w_1+w_2} \approx 10^{-20}$

in general: $$\underset{\text{outcome}\ \ \text{start}}{\pi_{ji}} = \frac{\Omega_{ji}}{\sum_{k\neq i} \Omega_{ki}} = \frac{\Omega_{ji}}{|\Omega_{ii}|}$$

3) recursion argument:

$\tau_i$ = avg. time $i \to i_c$ for first time

$$i \neq i_c: \qquad \underset{i\to i_c}{\tau_i} = \underset{\text{escape from } i}{\tau_i^{esc}} + \sum_{j\neq i} \underset{\text{prob. to land in } j}{\pi_{ji}}\ \underset{j\to i_c}{\tau_j} \qquad \tau_{i_c} = 0$$

$$= -\frac{1}{\Omega_{ii}} + \sum_{j\neq i} \frac{\Omega_{ji}}{(-\Omega_{ii})}\,\tau_j$$

multiply by $\Omega_{ii}$: $$\Omega_{ii}\tau_i = -1 - \sum_{j\neq i}\Omega_{ji}\tau_j$$

N equations for N unknowns $\tau_i$, $i=1,2,\dots,N$ $\Rightarrow$ $$\boxed{\sum_{j=1}^{N}\Omega_{ji}\tau_j = -1}$$ separate eqn. for each $i\neq i_c$

$$\boxed{\tau_{i_c} = 0}$$

example:

$$\begin{tikzcd} ① \arrow[r, bend left, "k_s"] & ② \arrow[l, bend left, "k_u"] \arrow[r, bend left, "k_b"] & ③ \end{tikzcd}$$

target: $i_c = 3$, $\tau_3 = 0$

$$\Omega = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} -k_s & k_u & 0 \\ k_s & -k_u-k_b & 0 \\ 0 & k_b & 0 \end{pmatrix} \end{array}$$

$i=1$: $$\sum_j \Omega_{j1}\tau_j = -1 \Rightarrow -k_s\tau_1 + k_s\tau_2 = -1$$

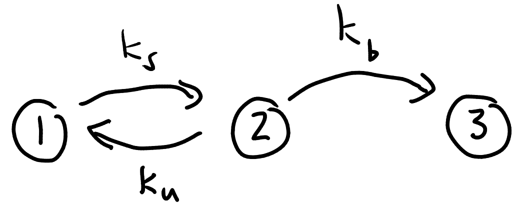
$i=2$: $$\sum_j \Omega_{j2}\tau_j = -1 \Rightarrow k_u\tau_1 - (k_u+k_b)\tau_2 + k_b\tau_3 = -1$$

$$\tau_3 = 0$$

$\Rightarrow$ solve: $$\tau_1 = \frac{k_s+k_u+k_b}{k_s k_b}$$ avg. time from 1 to 3 limits:

$\tau_1 \to 0$ if

$k_s, k_b \to \infty$




$\tau_1 \to \infty$ if $k_b \to 0$