LAST TIME: $\langle n_S n_E \rangle_t \approx \langle n_S \rangle_t \langle n_E \rangle_t$

define concentrations:

$$c_S(t) = \frac{\langle n_S \rangle_t}{V}$$

$$c_E(t) = \frac{\langle n_E \rangle_t}{V}$$

$\dots$

$$\Rightarrow \quad \frac{dc_S}{dt} = -\tilde{k}_b V c_S c_E + k_u c_{ES}$$

$$\frac{dc_E}{dt} = -\tilde{k}_b V c_S c_E + (k_u + k_{cat}) c_{ES}$$

4 eqn's $\Downarrow$ 4 unknowns

$$\frac{dc_{ES}}{dt} = \tilde{k}_b V c_S c_E - (k_u + k_{cat}) c_{ES}$$

$$\frac{dc_S^*}{dt} = k_{cat} c_{ES} \quad \Rightarrow \quad \text{Solve } c_S(t), \dots$$

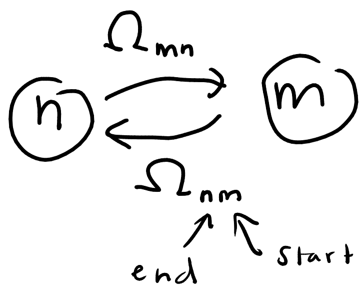
chemical kinetics eqn

or "law of mass action"

Next step: think about influence of energy

$\Rightarrow$ biological thermodynamics

system w/ transitions b/t states

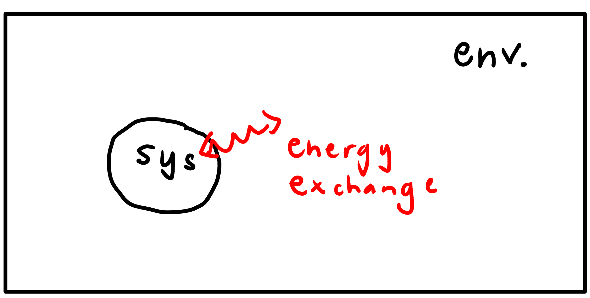


dynamics:

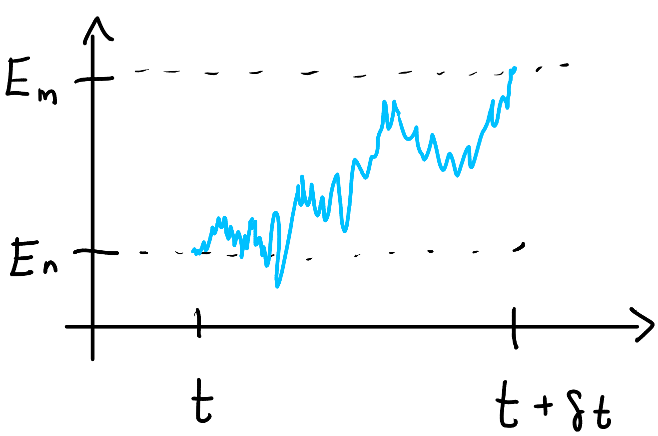
$$\frac{dp_n}{dt} = \sum_m \Omega_{nm} p_m$$

GOAL: connect Ω_{mn} to physical quantities (i.e. energy, chemical potentials, etc.)

know: state $n \Rightarrow$ energy E_n
state $m \Rightarrow$ energy E_m } internal energy of system



idea: Ω_{mn} (trans. $n \to m$) should be related to how likely it is to gain/lose $E_m - E_n$ energy from env.

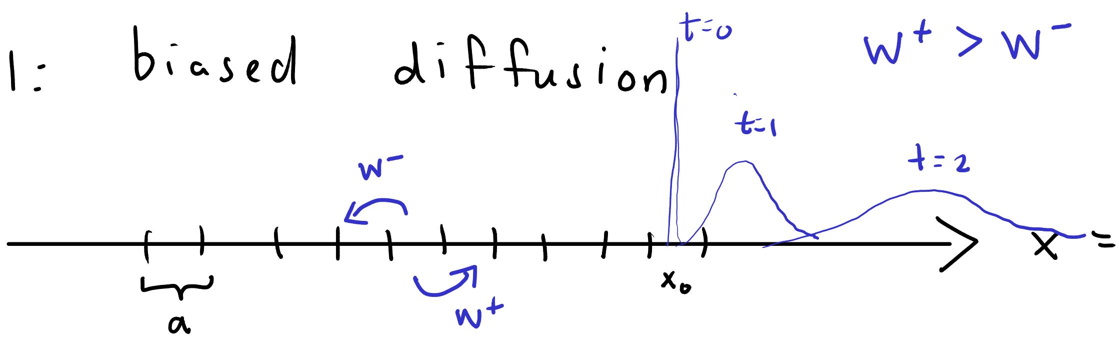


energy fluctuations:
biased random walk, w env. donating or removing energy in small increments

$\Omega_{mn}\,\delta t$ = prob for $n \to m$ trans. in time step δt, given start at state n

$\propto$ prob. system starting w/ energy E_n + ending up w/ energy E_m after time δt

recall: biased diffusion



continuum approx: (Fokker-Planck equ.)

$$\frac{\partial}{\partial t} p(x,t) = -v \frac{\partial}{\partial x} p + D \frac{\partial^2}{\partial x^2} p$$

$$v = (W^+ - W^-)a \qquad D = \frac{(W^+ + W^-)a^2}{2}$$

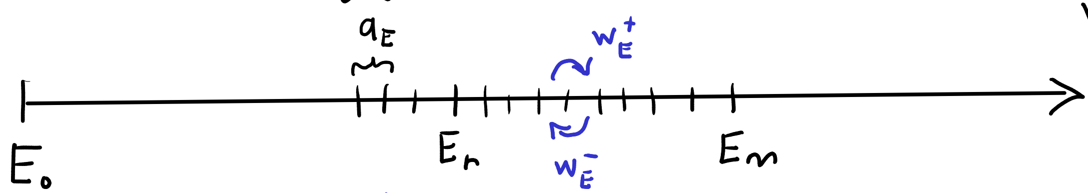
solution: $$p(x,t;x_0) = \frac{1}{\sqrt{4\pi D t}} \exp\left[-\frac{(x - x_0 - vt)^2}{4Dt}\right]$$

x_0: initial position

translate to energy space:

typically: $W_E^- > W_E^+$

$V_E < 0$



$a_E \sim$ energy (units)

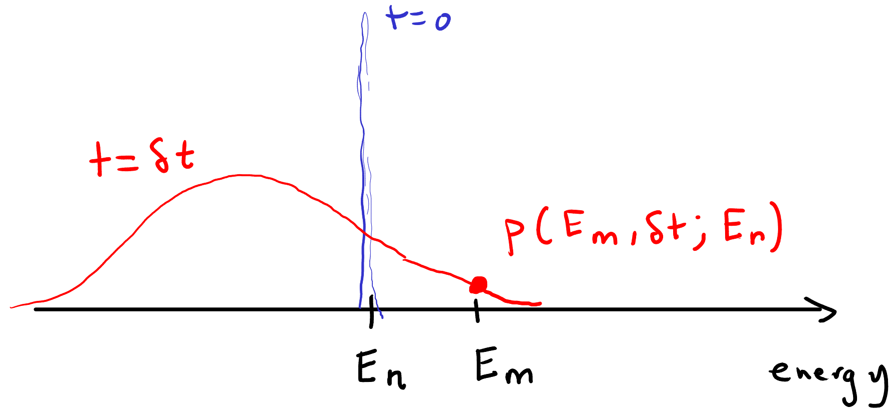
$W_E^+, W_E^- \sim \text{time}^{-1}$

$$V_E = (W_E^+ - W_E^-)\,a_E \qquad \frac{\text{energy}}{\text{time}}$$

$$D_E = \frac{1}{2}(W_E^+ + W_E^-)\,a_E^2 \qquad \frac{\text{energy}^2}{\text{time}}$$

$$p(E_m, \delta t; E_n) = \frac{1}{\sqrt{4\pi D_E \delta t}} \exp\left[-\frac{(E_m - E_n - V_E \delta t)^2}{4 D_E \delta t}\right]$$

prob. to end up at E_m after δt given start at E_n



$$\frac{p(E_m, \delta t; E_n)}{p(E_n, \delta t; E_m)} = \frac{\Omega_{mn}\delta t}{\Omega_{nm}\delta t} = \exp\left[\frac{\overset{<0}{V_E}}{\underset{>0}{D_E}}(E_m - E_n)\right]$$

$$= \exp\left[-\beta(E_m - E_n)\right]$$

$$\beta \equiv -\frac{V_E}{D_E} \equiv \frac{1}{k_B T}$$

T = temp. in Kelvin

$k_B = 1.38 \times 10^{-23}\ J/K$

fund. definition of $T = -\dfrac{D_E}{k_B V_E}$

typically $W_E^- > W_E^+ \Rightarrow V_E < 0 \Rightarrow T > 0$

bigger $|V_E| \Rightarrow T$ becomes smaller (env. is colder)