	quantum	master equ.
description of state	$\lvert\psi(t)\rangle$ vector in Hilbert space	$\vec{p}(t)$ vector in prob. space
dynamical equ.	$i\hbar\frac{\partial}{\partial t}\lvert\psi(t)\rangle = \hat{H}\lvert\psi(t)\rangle$	$\frac{\partial}{\partial t}\vec{p}(t) = \Omega\,\vec{p}(t)$
solution	$\lvert\psi(t)\rangle = e^{i\hat{H}t/\hbar}\lvert\psi(0)\rangle$	$\vec{p}(t) = e^{\Omega t}\vec{p}(0)$
avg. of observable	$A(t) = \langle\psi(t)\rvert\hat{A}\lvert\psi(t)\rangle$ $= \langle\psi(0)\rvert\hat{A}^H(t)\lvert\psi(0)\rangle$	$A(t) = \vec{A}\cdot\vec{p}(t)$ $= \vec{A}^H(t)\cdot\vec{p}(0)$ $\vec{A}^H(t) = \vec{A}^T e^{\Omega t}$
Heis. dyn. equation	$\frac{d}{dt}\hat{A}^H(t) = \frac{i}{\hbar}[\hat{H},\hat{A}^H(t)]$	$\frac{d}{dt}\vec{A}^H(t) = \vec{A}^H(t)\,\Omega$

observables in classical systems: state prop. A_n

vector $\vec{A} \Rightarrow$ components A_n

$$A(t) = \sum_n p_n(t)\,A_n = \vec{A}\cdot\vec{p}(t)$$

recall Heisenberg picture:

$$A(t) = \langle\psi(t)\rvert\hat{A}\lvert\psi(t)\rangle$$

$$= \langle\psi(0)\rvert\underbrace{\hat{U}^\dagger(t)\,\hat{A}\,\hat{U}(t)}_{\equiv \hat{A}^H(t)}\lvert\psi(0)\rangle$$

$\equiv \hat{A}^H(t)$ Heisenberg op. corresp. to $\hat{A}$

$$\lvert\psi(t)\rangle = \hat{U}(t)\lvert\psi(0)\rangle$$

$$\hat{U}(t) = e^{i\hat{H}t/\hbar}$$

$$\langle\psi(t)\rvert = \langle\psi(0)\rvert\hat{U}^\dagger(t)$$

"classical" Heisenberg picture:

$$A(t) = \sum_n A_n \, p_n(t)$$

$$= \sum_{n,m} A_n \left(e^{\Omega t}\right)_{nm} p_m(0)$$

$$= \sum_m A^H_m(t) \, p_m(0)$$

$$= \vec{A}^H(t) \cdot \vec{p}(0)$$

$$\vec{p}(t) = e^{\Omega t} \vec{p}(0)$$

$$p_n(t) = \left[e^{\Omega t} \vec{p}(0)\right]_n$$

$$= \sum_m \left(e^{\Omega t}\right)_{nm} p_m(0)$$

define:

$$A^H_m(t) \equiv \sum_n A_n \left(e^{\Omega t}\right)_{nm}$$

$$\vec{A}^H(t) \equiv \vec{A}^T e^{\Omega t}$$

interpreted as row vector

$$\frac{d}{dt} \vec{A}^H(t) = \vec{A}^T \frac{d}{dt} \underbrace{e^{\Omega t}}_{\hat{\mathbb{1}} + \Omega t + \frac{\Omega^2 t^2}{2!} + \dots} = \overbrace{\vec{A}^T e^{\Omega t}}^{\vec{A}^H(t)} \Omega$$

adjoint master equ.

"classical" Heis. dyn. equ: $\frac{d}{dt} \vec{A}^H(t) = \vec{A}^H(t) \, \Omega$

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payoff: write adjoint equ. in component form

$$\frac{d}{dt} A^H_n(t) = \sum_m A^H_m(t) \, \Omega_{mn}$$

$$\Omega_{nn} = -\sum_{m \neq n} \Omega_{mn}$$

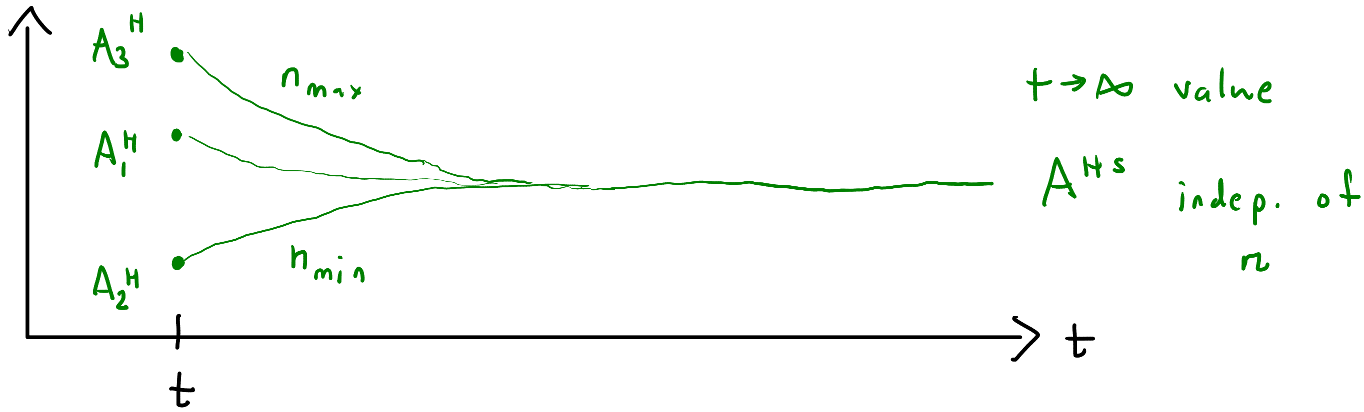
$$= \sum_{m \neq n} A^H_m(t) \, \Omega_{mn} + A^H_n(t) \left(-\sum_{m \neq n} \Omega_{mn}\right)$$

$$\boxed{\frac{d}{dt} A^H_n(t) = \sum_{m \neq n} \left[A^H_m(t) - A^H_n(t)\right] \Omega_{mn}}$$

consider any time t along trajectory:

$n = n_{max}$ has the largest $A_n^H(t)$

$n = n_{min}$ has the smallest $A_n^H(t)$



$n_{max} = 3$

$n_{min} = 2$

$$\frac{d}{dt} A^H_{n_{max}} = \sum_{m \neq n_{max}} \underbrace{\left(A_m^H - A^H_{n_{max}}\right)}_{< 0} \underbrace{\Omega_{m n_{max}}}_{\text{at least one } m \text{ exists where } \Omega_{m n_{max}} > 0 \text{ (b/c graph is connected)}}$$

< 0 top curve always decreases

$$\frac{d}{dt} A^H_{n_{min}} = \sum_{m \neq n_{min}} \underbrace{\left(A_m^H - A^H_{n_{min}}\right)}_{> 0} \underbrace{\Omega_{m n_{min}}}_{\text{at least one } m \text{ where } > 0}$$

> 0 bottom curve always increases

$$A_n^H(t) \xrightarrow[t \to \infty]{} A^{Hs} \text{ for any } n$$

true for <u>any</u> observable $\vec{A}$ & corr. $\vec{A}^H(t)$

create a fake observable labeled by $k = 1, \dots, N$

$$\vec{A}^{(k)} \Rightarrow A_n^{(k)} \equiv \delta_{nk} = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$

vector   comp.

↳ unit vector w/ 1 in k th pos., zeros elsewhere ⇒ "indicator" observable

$$A_n^{(k)H}(t) \xrightarrow[t\to\infty]{} A^{(k)Hs}$$

constant corr. to kth indicator observable

average associated w/ k th indicator observable

$$A^{(k)}(t) = \sum_n A_n^{(k)} p_n(t) = p_k(t)$$

$$p_k(t) = \sum_m \underbrace{A_m^{(k)H}(t)}_{\to \text{ const. } A^{(k)Hs} \text{ as } t\to\infty} p_m(0)$$

$$\Rightarrow \quad p_k(t) \xrightarrow[t\to\infty]{} p_k^s$$

const. limiting value as $t \to \infty$

for const. Ω + connected graph: sys. will go to a stationary state