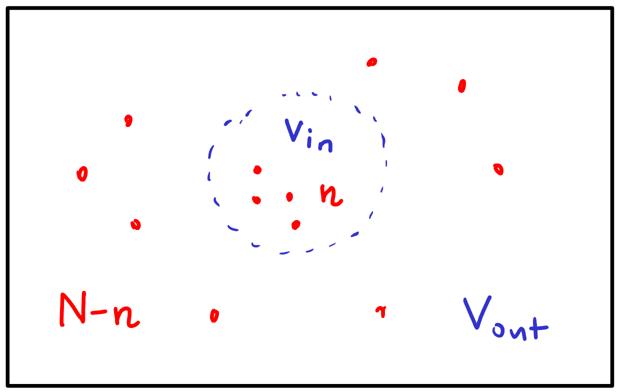
Transport across membranes:

I. Uncharged molecules



vol. $V_{tot} = V_{in} + V_{out}$

for simplicity: V_{in}, V_{out}, N fixed

model: state n

$\Rightarrow$ n molec. inside

$N-n$ molec. outside

$$\begin{tikzcd} \cdots & n-1 \arrow[r, bend left, "k_{in}(n-1)"] & n \arrow[l, bend left, "k_{out}(n)"] \arrow[r, bend left, "k_{in}(n)"] & n+1 \arrow[l, bend left, "k_{out}(n+1)"] & \cdots \end{tikzcd}$$

no enthalpy diff. b/t states

LDB: $$\frac{k_{out}(n+1)}{k_{in}(n)} = e^{-\beta\left(\mu_{out}(n+1) - \mu_{in}(n)\right)}$$

$$\mu_{out}(n) = \mu_0 + k_B T \ln \overbrace{c_{out}(n)}^{\frac{N-n}{V_{out}}}$$

$$\mu_{in}(n) = \mu_0 + k_B T \ln \underbrace{c_{in}(n)}_{n/V_{in}}$$

$$\frac{k_{out}(n+1)}{k_{in}(n)} = \frac{c_{in}(n)}{c_{out}(n+1)}$$

$n \gg 1$

$k_{out}(n+1) \approx k_{out}(n)$

$$\Rightarrow \frac{k_{out}(n)}{k_{in}(n)} \approx \frac{c_{in}(n)}{c_{out}(n)}$$

net rate of molec. leaving cell (V_{in}):

$$F(n) = k_{out}(n) - k_{in}(n)$$

$$= k_{out}(n)\left(1 - \frac{k_{in}(n)}{k_{out}(n)}\right)$$

$$= k_{out}(n)\left(1 - \frac{c_{out}(n)}{c_{in}(n)}\right)$$

$$= \frac{k_{out}(n)}{c_{in}(n)}\left(c_{in}(n) - c_{out}(n)\right)$$

$F = 0$ (equil.) occurs when $c_{in}(n_0) = c_{out}(n_0)$

$$\frac{n_0}{V_{in}} = \frac{N - n_0}{V_{out}}$$

$$n_0 = N\frac{V_{in}}{V_{in} + V_{out}}$$

$$= N\frac{V_{in}}{V_{tot}}$$

Taylor expand $F(n)$ around $n = n_0$:

$$k_0 = k_{out}(n_0)$$

$$c_0 = \frac{N}{V_{tot}}$$

$$\frac{k_{out}(n)}{c_{in}(n)} \approx \frac{k_{out}(n_0)}{c_{in}(n_0)} + \ldots$$

$$= \frac{k_0}{c_0} + \ldots$$

$$c_{in}(n) - c_{out}(n) \approx \underbrace{O(n - n_0)}_{\text{1st order term}} + \ldots$$

$$F(n) \approx \frac{k_0}{c_0}\left(c_{in}(n) - c_{out}(n)\right)$$

accurate up to $\underline{1}$st order in $n - n_0$

net flux of particles leaving

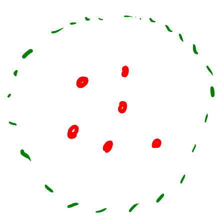
$$\frac{F(n)}{A} = \underbrace{\frac{k_0}{c_0 A}}\left(c_{in}(n) - c_{out}(n)\right)$$

$\uparrow$ membrane area

$$\rho = \frac{k_0}{c_0 A} = \frac{s^{-1}}{nm^{-3}\, nm^2} = \frac{nm}{s}$$

permeability coeff.

rough estimates of how quickly things move out of cells:



$V_{out} \to \infty$

$c_{out} \to 0$

$n_0 \to 0$

dynamics of n on avg. (ignore stoch. fluctuations)

$$\frac{dn}{dt} \approx -F(n) = -\rho A\, c_{in}(n)$$

divide by V_{in}

$$\frac{dc_{in}}{dt} = -\frac{\rho A}{V_{in}} c_{in}$$

$$c_{in}(t) = c_{in}(0)\, e^{-t/\tau}$$

charact. time for things leaving cell

$$\tau = \frac{V_{in}}{\rho A} = s$$

V_{in}: nm^3 $\quad$ ρ: nm/s $\quad$ A: nm^2

Na^+ : $\rho \sim 10^{-5}$ nm/s $\qquad A = 4\pi R^2$

$R \sim 1000$ nm cell radius $\qquad V_{in} = \frac{4}{3}\pi R^3$

$\Rightarrow \tau \sim 3 \times 10^7$ s > 1 yr

H_2O ; $\rho \sim 10^5$ nm/s

$\Rightarrow \tau \sim 3 \times 10^{-3}$ s