From last time: cell membrane as a capacitor

$$\rho = \frac{Q}{A} \quad \frac{\text{charge}}{\text{area}} \qquad V = \frac{\rho d}{\varepsilon}$$

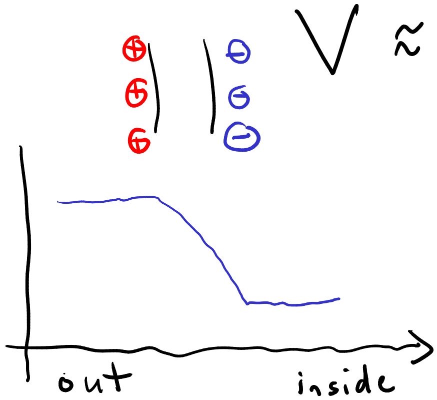
d = separation of plates (≈ 4 nm)

$$C = \frac{Q}{V} = \frac{\varepsilon A}{d}$$

ε = permittivity $= \varepsilon_r \varepsilon_0$
$\varepsilon_r \downarrow 2$

specific capacitance $\frac{C}{A} = \frac{\varepsilon}{d} \sim 1 \frac{\mu F}{cm^2}$

typical voltage across membranes:



$V \approx -50$ to -200 mV

measured from outside to inside

let's say $V = -100$ mV

cell area $A \sim 3\ \mu m^2$

$$\frac{Q}{e} = \frac{CV}{e} = 2 \times 10^4$$

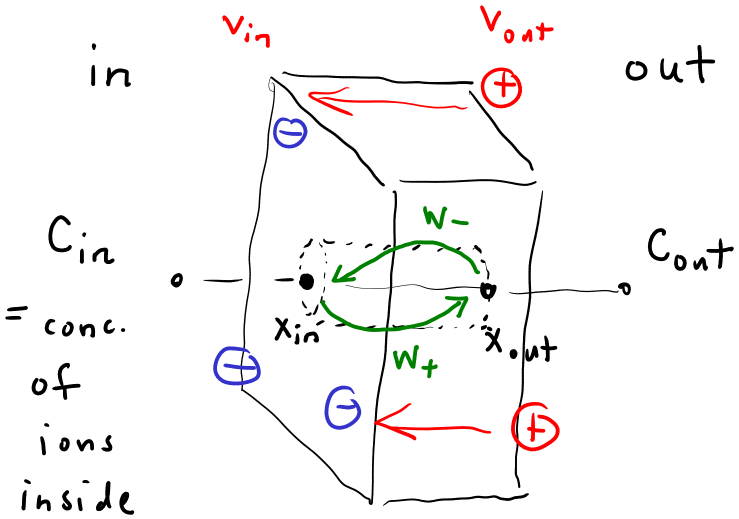
total charges on surface

total ion conc. ~ 100 mM

$\approx 10^8$ charges per fL

bacterial cell ≈ 1 fL

transport of ions through channels in membrane:



$$\Delta V = V_{out} - V_{in}$$

one type of ion w/ some charge q

$$\frac{W_+}{W_-} = e^{-\beta\left(\underbrace{E_{out} - E_{in}}_{-q\Delta V} + \underbrace{k_B T \ln \frac{C_{out}}{C_{in}}}_{\text{chem. pot. diff.}}\right)}$$

due to moving ions against a potential

$$\Delta V = -100 \text{ mV}$$

$$|e\Delta V| \approx 4\, k_B T$$

when term in ( ) = 0 $\Rightarrow$ $W_+ = W_-$

no net charge current

$$-q\,\Delta V + k_B T \ln \frac{C_{out}}{C_{in}} = 0$$

equil. $\Delta V = \dfrac{k_B T}{q} \ln \dfrac{C_{out}}{C_{in}} \equiv V_N$ Nernst potential

rewrite detailed balance equ.:

$$\frac{W_+}{W_-} = e^{-\beta q (V_N - \Delta V)}$$

if $\Delta V > V_N \Rightarrow W_+ > W_-$ net flow out

$\Delta V < V_N \Rightarrow W_+ < W_-$ net flow in

imagine we start $\Delta V > V_N$ $q = +e$

net flow out $\Rightarrow$ more excess pos. ions on outside

ΔV becomes more negative (decreases)

C_{in} + C_{out} $\approx$ same (b/c small flow of charges does not change conc. a lot)

$\Rightarrow$ eventually ΔV reaches V_N

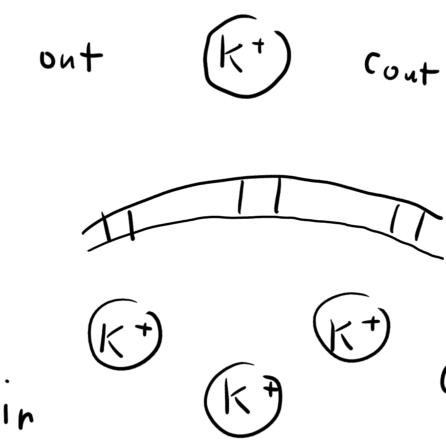
$\Rightarrow$ flow stops

same argument when $\Delta V < V_N$

$\Rightarrow$ ΔV increases until flow stops

concrete example: focus on one ion type

K^+ $q = +e$



$c_{out} = 20$ mM

$C_{in} = 400$ mM

$$V_N = \frac{k_B T}{q} \ln \frac{C_{out}}{C_{in}} = -75 \text{ mV}$$

typical $\Delta V = -65$ mV

net current out of cell $I \propto W_+ - W_-$

Taylor expand $I \approx G(\Delta V - V_N)$

$+ \ldots$

$$\frac{\text{current}}{\text{area}} = j = \frac{I}{A}$$

$$= g(\Delta V - V_N)$$

membrane conductivity

$$g = \frac{\sigma}{A}$$

$$\approx 10\ \Omega^{-1}\,m^{-2}$$

for K^+ channels in squid axons

reality: multiple ion types, each w/ specialized channels

for type i: $j_i = g_i(\Delta V - \underbrace{V_N^{(i)}}_{\frac{k_B T}{q_i} \ln \frac{c_{out}^{(i)}}{c_{in}^{(i)}}})$

g_i: depends on # open channels / area

ΔV: global const. b/c same membrane

two most important ions: K^+, Na^+ $(q = +e)$

	c_{in}	c_{out}	$V_N^{(i)}$
K^+	400 mM	20 mM	−75 mV
Na^+	50 mM	440 mM	54 mV

to get equilibrium (no current flows)

at $\Delta V = -65\text{ mV} = V_0$ (equil. memb. potential) need pumping of Na^+ + K^+ ions

$$0 = j_{Na} = g_{Na}\underbrace{(V_0 - V_N^{Na})}_{<0} + \underset{>0}{j_{Na}^{pump}}$$

$$0 = j_K = g_K\underbrace{(V_0 - V_N^{K})}_{>0} + \underset{<0}{j_K^{pump}}$$

in addition to passive channels

we need Na pumped out + K pumped in