Potential of cell membrane: $V_0 = -65$ mV

equilibrium (Nernst) potentials:

$$V_N^{Na} = +54 \text{ mV}$$
$$V_N^{K} = -75 \text{ mV}$$

in order for cell to achieve equil:

$$j_{Na}^{pump} > 0 \quad \text{(pumping Na out)}$$
$$j_{K}^{pump} < 0 \quad \text{(pumping K in)}$$

solution: Na/K pump takes 3 Na^+ out + 2 K^+ in for every cycle, consuming 1 ATP

Neuron / kidney cells: 50-70% of total ATP consumption is used by these pumps

equil:

$$0 = j_{Na} = g_{Na}(V_0 - V_N^{Na}) + j_{Na}^{pump}$$
$$0 = j_K = g_K(V_0 - V_N^{K}) + j_K^{pump}$$
$$j_K^{pump} = -\frac{2}{3} j_{Na}^{pump}$$

solve for $V_0 = \dfrac{2 g_{Na} V_N^{Na} + 3 g_k V_N^K}{2 g_{Na} + 3 g_K}$

$V_N^{Na} = 54 \text{ mV}$ $\quad V_N^k = -75 \text{ mV}$

norming resting state: $g_K >> g_{Na}$

(most Na channels closed)

$g_K = 25\, g_{Na}$

$\Rightarrow$ V_0 close to V_N^K

$\Rightarrow$ $V_0 \approx -65 \text{ mV}$

What if memb. potential $\Delta V = V_0 + u$

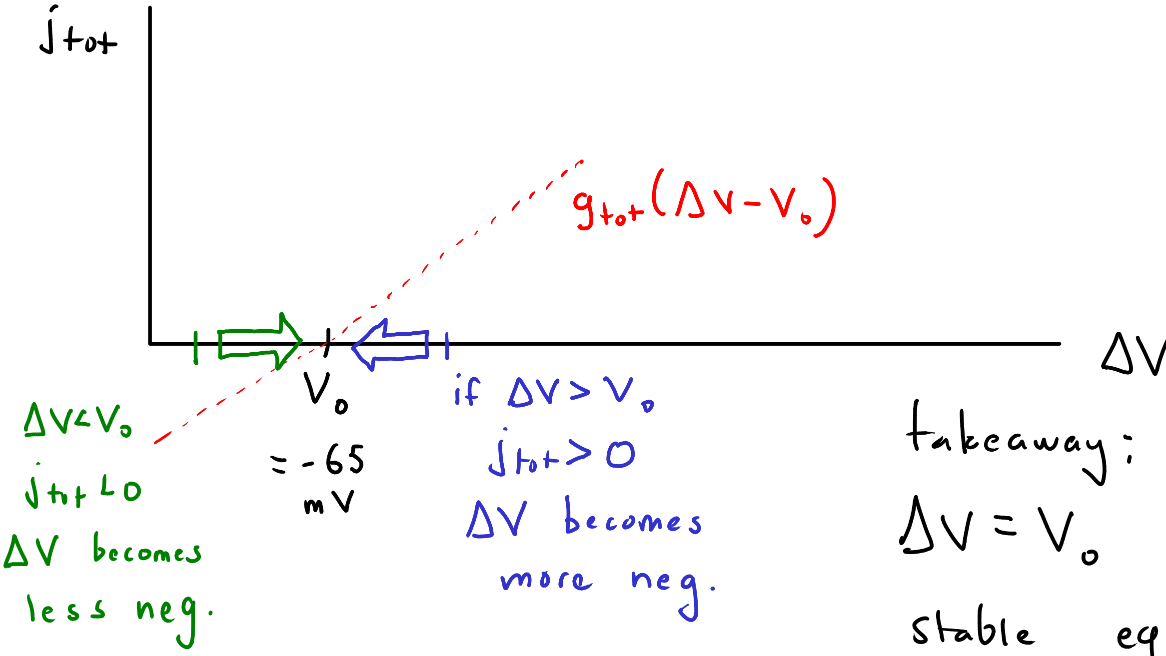
perturbation

$u > 0 \Rightarrow$ putting more $-$ charges outside relative to resting state

$$\dot{j}_{tot} = \dot{j}_K + \dot{j}_{Na} = g_K (u + V_0 - V_N^k) + \dot{j}_K^{pump}$$

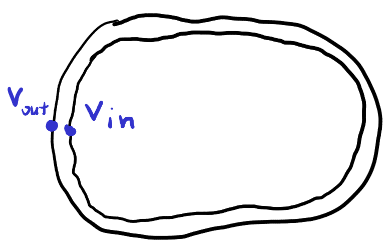
$$+ \; g_{Na} (u + V_0 - V_N^{Na}) + \dot{j}_{Na}^{pump}$$

$$= \underbrace{(g_K + g_{Na})}_{g_{tot}} u \qquad u \equiv \Delta V - V_0$$



takeaway:
$\Delta V = V_0$ is a stable equil. point

Build circuit model of cell:



$V = V_{in} - V_{out}$

A_{mem} = memb. area

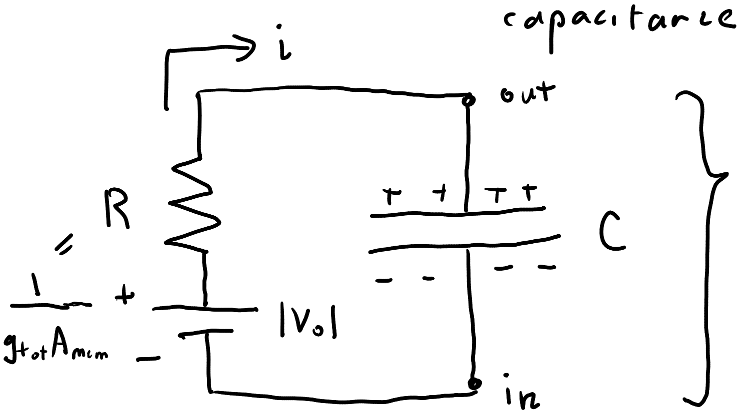
$V_0 = -65\,mV$

$C_s = \dfrac{\text{capac.}}{\text{area}}$

$C = C_s A_{mem}$ total capacitance

$$i = j_{tot} A_{mem} = -C \frac{dV}{dt} = \underbrace{g_{tot} A_{mem}}_{R^{-1}} (V - V_0)$$

$$\frac{dV}{dt} = -\frac{1}{RC}(V - V_0)$$



$V = V_{in} - V_{out}$

$$V(t) = V_0 + e^{-t/RC}(V_{init} - V_0)$$

$t = 0 \quad V(t) = V_{init}$

$t \to \infty \quad V(t) \to V_0$

$t \gg RC$

$$RC = \frac{C_s A_{mem}}{g_{tot} A_{mem}} = \frac{C_s}{g_{tot}}$$

$$C_s \sim 1 \ \mu F/cm^2 \qquad g_{tot} \sim 10 \ \Omega^{-1} m^{-2}$$

$$RC = \frac{C_s}{g_{tot}} \sim 1 \ ms$$

What is missing: V is not really the same across whole cell membrane for large cells (like neurons w/ really long axons)