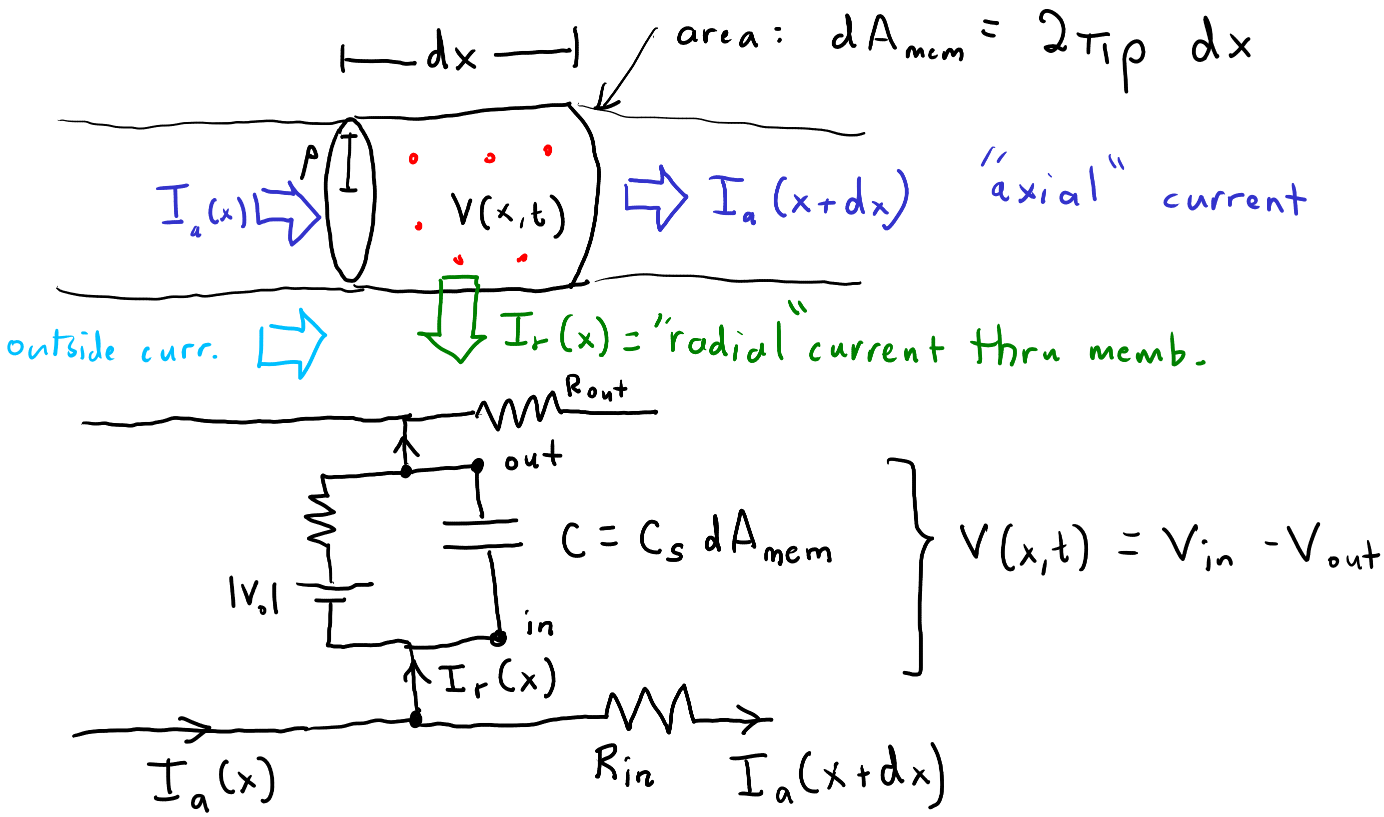
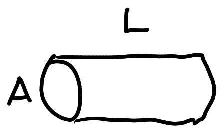


Pouillet's law: resistance of cylindrical wire



σ = conductivity (material prop.)

$$R = \frac{L}{\sigma A}$$

$$R_{in} = \frac{dx}{K \pi \rho^2}$$

K = conductivity of inside of axon

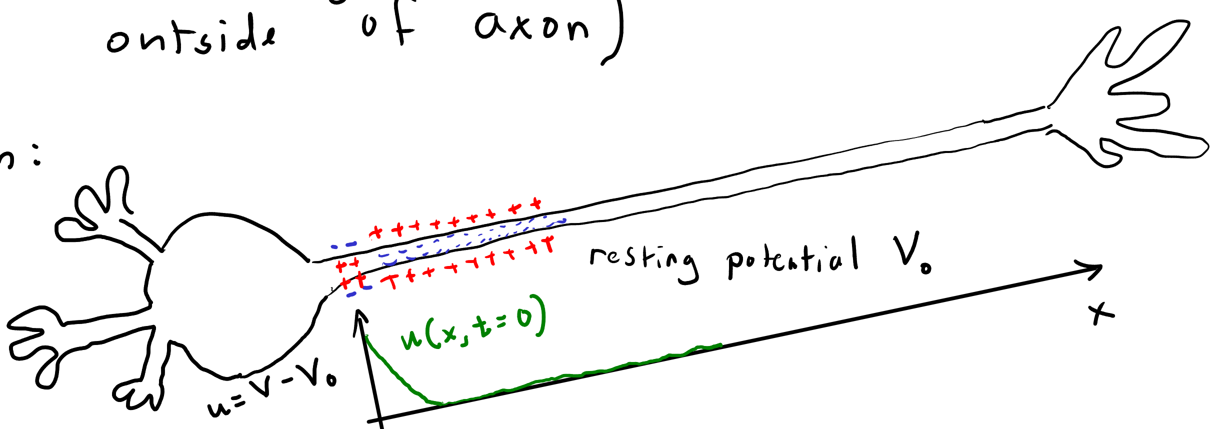
$\propto$ conc. of total ions

$\sim 3\ \Omega^{-1} m^{-1}$

$R_{out} \approx 0$

(A is huge for outside of axon)

Problem:



Question: how does spike $u(x,t)$ propagate?
how fast does it move?

potential change inside axon across one dx segment:

$$dV = -I_a(x) R_{in}$$

$$= -I_a(x) \frac{dx}{K\pi\rho^2}$$

$$\Rightarrow \quad I_a(x) = -K\pi\rho^2 \frac{dV}{dx} \qquad (1)$$

current conservation:

$$\underbrace{I_a(x) - I_a(x+dx)}_{\approx -dx \frac{dI_a}{dx}} = \underbrace{I_r(x)}_{dA_{mem}\, g_{tot}(V-V_0) + dA_{mem} C_s \frac{dV}{dt}}$$

$$\Rightarrow \quad -dx \frac{dI_a}{dx} = 2\pi\rho \, dx \left[g_{tot}(V-V_0) + C_s \frac{dV}{dt} \right] \qquad (2)$$

plug (1) into (2), divide by dx:

$$K\pi\rho^2 \frac{d^2V}{dx^2} = 2\pi\rho \left[\underbrace{g_{tot}(V-V_0)}_{j_{tot}(V)} + C_s \frac{dV}{dt} \right]$$

$= $ total ion current thru mem. channels

$$u(x,t) = V(x,t) - V_0$$

$$\lambda \equiv \sqrt{\frac{\rho \kappa}{2 g_{tot}}} \quad \text{[units: length]}$$

$$\tau = \frac{C_s}{g_{tot}} \quad \text{[units: time]}$$

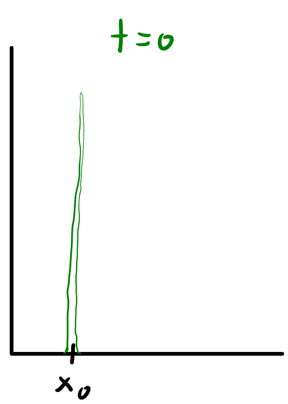
$$\boxed{\lambda^2 \frac{\partial^2 u}{\partial x^2} - \tau \frac{\partial u}{\partial t} = u}$$

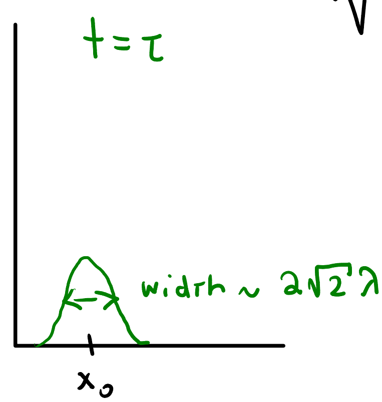
cable equation

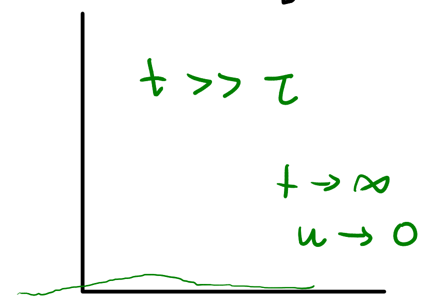
trivial sol'n: $u(x,t) = 0$ everything at resting potential

initial spike: $u(x,0) = B\,\delta(x - x_0)$

Solution: $$u(x,t) = \frac{e^{-t/\tau}}{\sqrt{4\pi t \lambda^2 \tau^{-1}}} \exp\left[\frac{-(x-x_0)^2}{4 + \lambda^2 \tau^{-1}}\right]$$







speed of spreading out $\dfrac{\text{dist}}{\text{time}} \sim \dfrac{\lambda}{\tau} \sim 9$ m/s

for squid:

$\rho = 0.5$ mm

$\kappa = 3\ \Omega^{-1}\ m^{-1}$

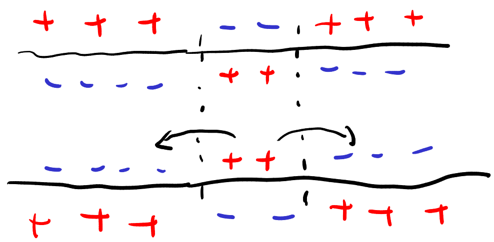
$g_{tot} = 10\ \Omega^{-1}\ m^{-2}$

$C_s = 1\ \mu F/cm^2$

$\lambda = 9$ mm

$\tau = 1$ ms

problem: decay of potential



excess charge spreads + is then leaked / pumped out

$\Rightarrow$ potential returns to V_0

$\Rightarrow$ $u(x, t\to\infty) \to 0$

revisit: $j_{tot}(V) = g_{tot}(V - V_0)$

derived under assumption most Na channels were closed

$$V_0 = \frac{2 g_{Na} \overset{54\text{ mV}}{V_N^{Na}} + 3 g_K \overset{-75\text{ mV}}{V_N^K}}{2 g_{Na} + 3 g_K}$$

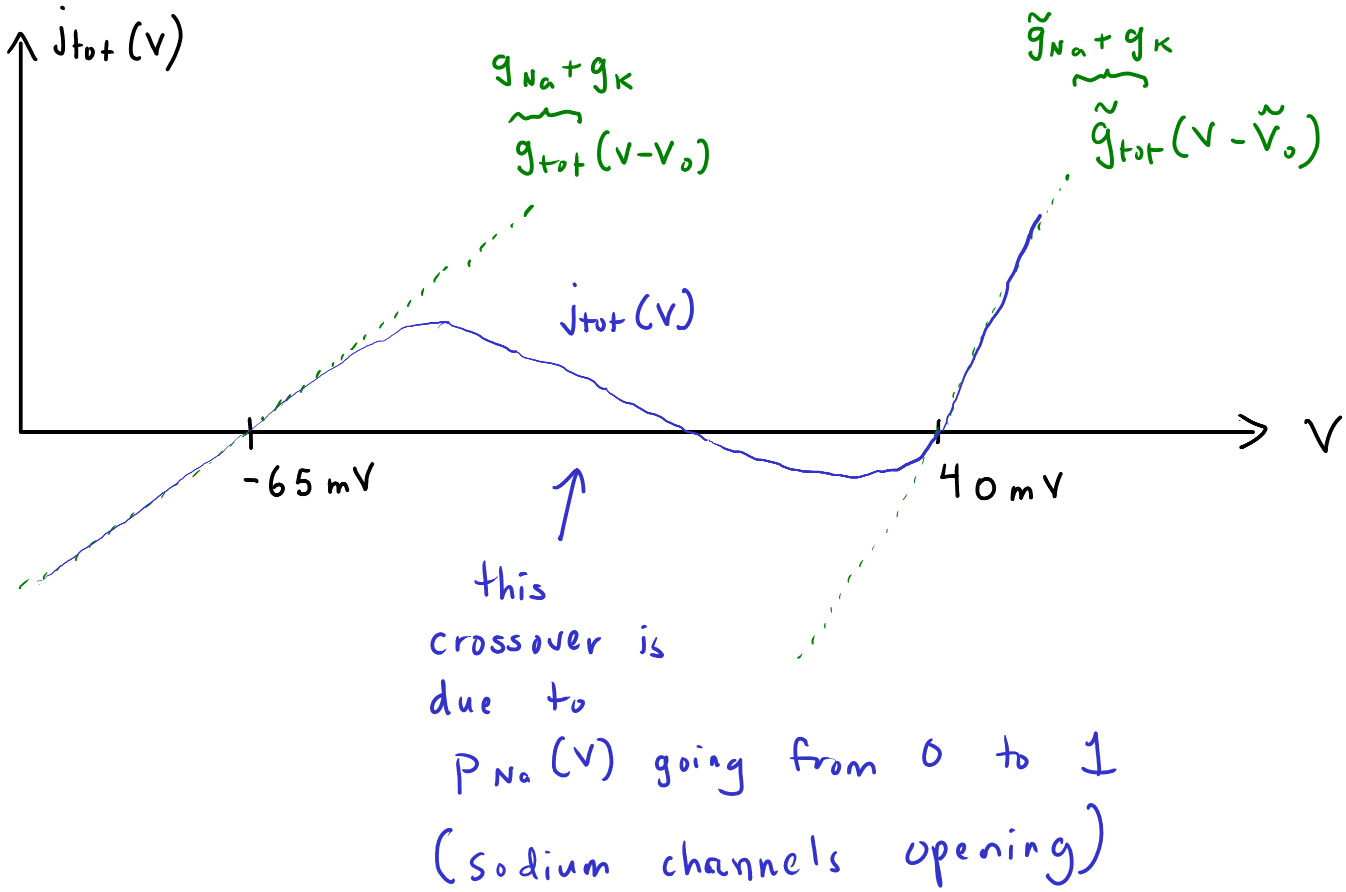
Na channels closed: $g_{Na} \ll g_K$

$V_0 = -65$ mV

if Na channels were open:

new value $\tilde{g}_{Na} \gg g_K$ (more Na channels than K)

$\Rightarrow$ $\tilde{V}_0 = 40$ mV closer to V_N^{Na}



next time: cable eqn w/ nonlinear $j_{tot}(V)$