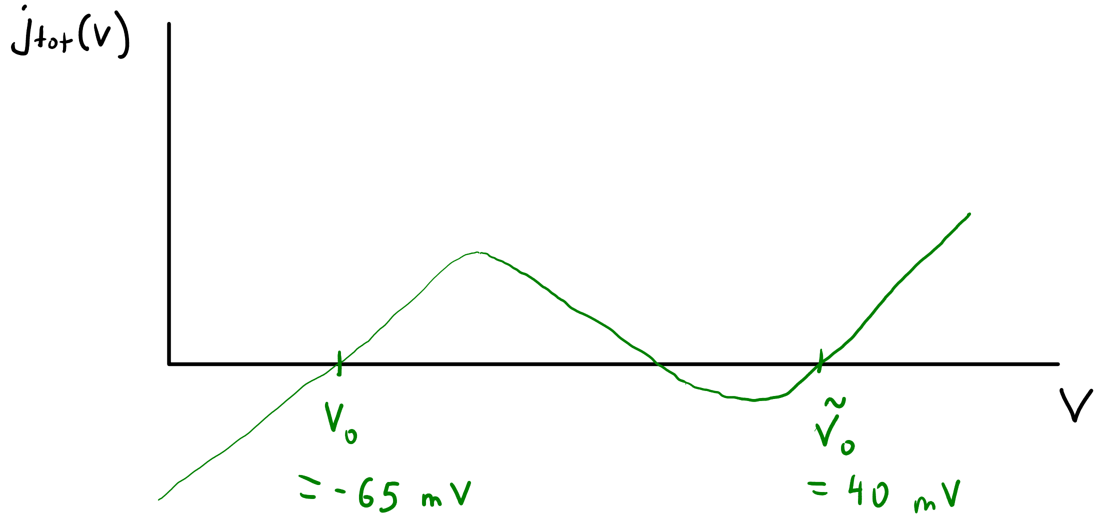


nonlinear cable equation:

$$\pi \rho^2 K \frac{\partial^2 V}{\partial x^2} = 2\pi\rho \left[j_{tot}(V) + C_s \frac{\partial V}{\partial t} \right]$$

$$u(x,t) = V(x,t) - V_0 \qquad \lambda = \sqrt{\frac{\rho K}{2 g_{tot}}} \qquad \tau = \frac{C_s}{g_{tot}}$$

$$\Rightarrow \quad \lambda^2 \frac{\partial^2 u}{\partial x^2} - \tau \frac{\partial u}{\partial t} = f(u) \qquad f(u) = \frac{j_{tot}(V_0 + u)}{g_{tot}}$$

previously: $f(u) = u$

guess a traveling wave solution:

<u>soliton</u> = traveling wavefront that
maintains its shape
b/c a balance of dissipation
(ions leaking thru membrane)
+ nonlinear reinforcement
(opening of Na channels)

$$u(x,t) = (\tilde{V}_0 - V_0)\, \underset{\substack{\uparrow \\ \text{unknown function}}}{w}\left(\underbrace{\frac{x - st}{\lambda}}_{\equiv\, \theta \ \text{dimensionless}}\right)$$

$s =$ speed of wave

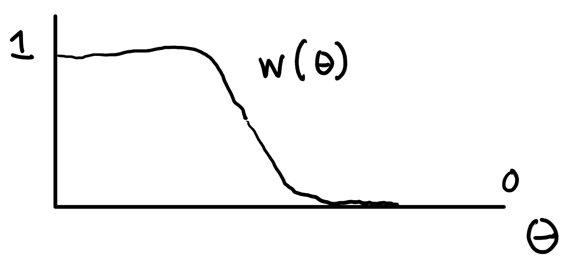
w has the properties:

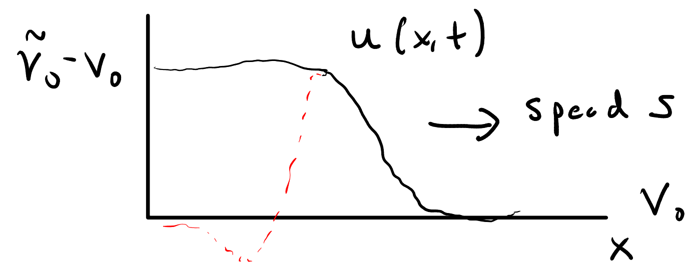
$$w(\theta \to -\infty) = 1$$

$t \to +\infty \Rightarrow u = \tilde{V}_0 - V_0$ (all sodium channels open)

$$w(\theta \to +\infty) = 0$$

$t \to -\infty \Rightarrow u = 0$ (Na channels closed)





plug guess into cable equation:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\lambda^2}(\tilde{V}_0 - V_0)\frac{\partial^2 w}{\partial \theta^2}$$

$$\frac{\partial u}{\partial t} = -\frac{s}{\lambda}(\tilde{V}_0 - V_0)\frac{\partial w}{\partial \theta}$$

$$\Rightarrow \quad \frac{\partial^2 w}{\partial \theta^2} = -\underbrace{\frac{\tau s}{\lambda}}_{\gamma}\frac{\partial w}{\partial \theta} + \underbrace{F(w)}_{\equiv \frac{f(w)}{\tilde{V}_0 - V_0}}$$

$w(\theta)$

$\gamma \sim$ "friction coeff."

this looks like a classical mech. problem of a particle:

$\theta \sim$ "time" $\qquad$ mass $m \sim 1$

$w(\theta) \sim$ "distance"

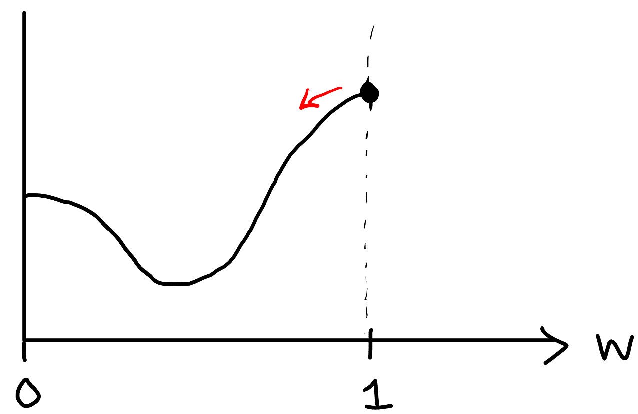
$\dfrac{\partial w}{\partial \theta} \sim$ "velocity"

$-\gamma \dfrac{\partial w}{\partial \theta} \sim$ "drag friction force"

$F(w) \sim$ "force"

$$F(w) = -\Phi'(w)$$

"potential" $\quad \Phi(w) = -\displaystyle\int_0^w dw' \, F(w')$



claim: there exists unique value $\gamma = \gamma_0$ where the ball will start near $w=1$ at $t \to -\infty$ & end up at $w=0$ at $t \to \infty$

$$\gamma = \frac{\tau s}{\lambda} = \overset{\text{const.}}{\gamma_0}$$

$$s = \underbrace{\frac{\lambda}{\tau}}_{9\ m/s} \gamma_0$$

$\gamma_0 \to$ some dimensionless number $\sim O(1)$