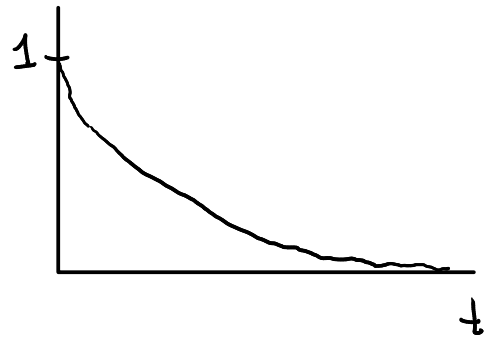


$$U_c(n) = \left(1 - \frac{1}{2N}\right)^n$$

$$t = nt_g$$

$$U_c(t) \approx \exp\left(-\frac{t}{2Nt_g}\right)$$



prob. two lineages
have not merged going
+ in the past

WF model but w/ time-varying population
 $\tilde{N}(t)$:

$$U_c(t) = \left(1 - \frac{1}{2\tilde{N}(-t_g)}\right) \cdot \left(1 - \frac{1}{2\tilde{N}(-2t_g)}\right)$$

$$\dots \left(1 - \frac{1}{2\tilde{N}(-nt_g)}\right)$$

$$1 - x$$

$$\approx \exp(-x)$$

$$\approx \exp\left[-\frac{1}{t_g} \sum_{j=1}^n t_g \frac{1}{2\tilde{N}(-jt_g)}\right] \quad t = nt_g$$

$$U_c(t) \approx \exp\left[-\frac{1}{t_g} \int_0^t dt' \frac{1}{2\tilde{N}(t')}\right]$$

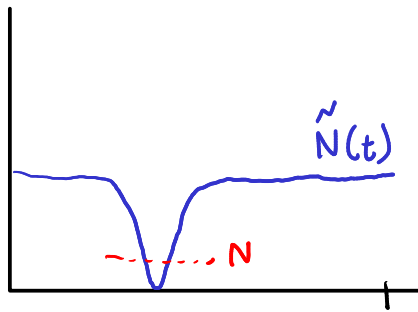
compare to fixed model:

$$U_c(t) \approx \exp\left[-\frac{t}{2Nt_g}\right]$$

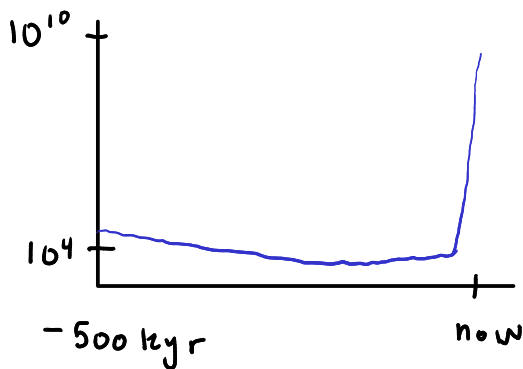
map onto fixed N model w/ eff. pop.

$$N = \left[\frac{1}{t} \int_0^t dt' \frac{1}{\tilde{N}(t')} \right]^{-1}$$

$$\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots}$$



this is dominated by times in the past where populations are small



On average, when did coalescence occur?

$P_c(t) dt \equiv$ prob. that coal. occurred
b/t t + $t+dt$

$$= U_c(t) - U_c(t+dt)$$

$$\approx -dt \frac{dU_c}{dt} \quad P_c(t) = -\frac{dU_c}{dt}$$

mean coalesc. time $t_c = \int_0^\infty dt \, t \, P_c(t)$

$$= - \int_0^\infty dt \, t \, \frac{dU_c}{dt}$$

$$= - \int_0^{\infty} dt \quad t \left(-\frac{1}{2Nt_g} \right) e^{-\frac{t}{2Nt_g}}$$

$$\Rightarrow t_c = 2N t_g$$

$\uparrow$ $\uparrow$
 10^4 20 yrs

$$\approx 400 \text{ kyr}$$

rough
agreement
w/ better
estimates

mitochondrial Eve $\sim 99-152 \text{ kyr}$

Y-chromosome Adam $\sim 120-581 \text{ kyr}$

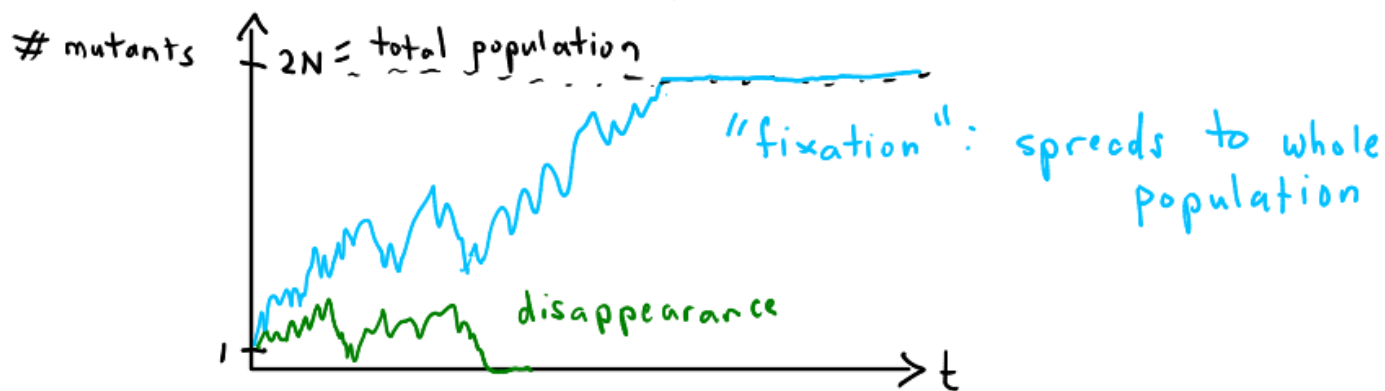
$$H = \frac{\theta}{1+\theta}$$

$$\theta = 4N\mu$$

males or
females only

$$\theta = 2N\mu$$

Dynamics of mutations



Assume mutation rate μ is small: a mutant appears rarely & when it does it disappears or fixes before a new mutant appears.

Want to calculate:

$$\text{substitution rate} = k = \underbrace{2N\mu}_{\substack{\text{rate at which new} \\ \text{mutations get fixed} \\ \text{in the population}}} \cdot \underbrace{\pi_{\text{fix}}}_{\substack{\text{prob that} \\ \text{mutation} \\ \text{gets fixed}}}$$