Papers: Lande, PNAS 1981
Kirkpatrick, Evolution 1982

2 genes, → present in both males + females

2 alleles:

T_1 : no ornament
T_2 : ornament
} only expressed in males

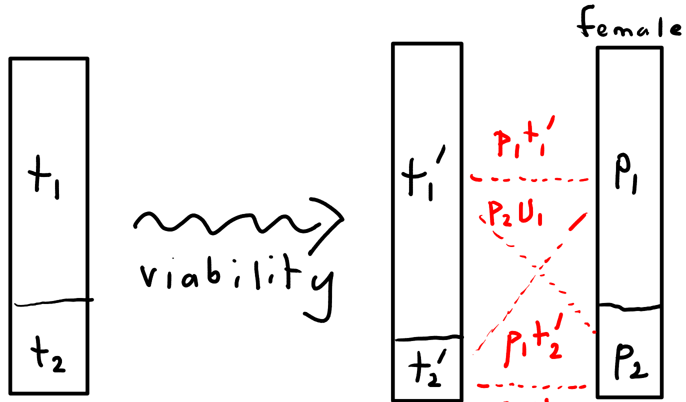
P_1 : no pref. for ornament
P_2 : pref.
} only expressed in females

Combos: $T_1 P_1$, $T_1 P_2$, $T_2 P_1$, $T_2 P_2$

$t_1 = 1 - t_2$ $\quad t_2$ = frac. of males w/ ornament = males $T_2 P_1 + T_2 P_2$

$p_1 = 1 - p_2$ $\quad p_2$ = frac. of females w/ pref. = females $T_1 P_2 + T_2 P_2$

$0 \le t_2 \le 1 \qquad 0 \le p_2 \le 1$



male pop. at current time

pairings

pref. parameter: $\beta \ge 1$

$$U_2 = \frac{\beta t_2'}{t_1' + \beta t_2'}$$

$\beta \to \infty$: $U_2 \to 1$

$$U_1 = \frac{t_1'}{t_1' + \beta t_2'}$$

$\beta \to 1$: no pref. $U_2 = t_2'$, $U_1 = t_1'$

fitness penalty for ornament:

certain death $\quad$ no penalty

$0 \le \alpha \le 1$

$$t_2' = \frac{\alpha t_2}{t_1 + \alpha t_2} \qquad t_1' = \frac{t_1}{t_1 + \alpha t_2}$$

$\alpha = 1$: $t_1' = t_1 \quad t_2' = t_2$

male relative fitness $\sim \dfrac{\text{frac. of pairings}}{\text{frac. of population}} = \dfrac{V_i}{t_i}$

$$V_2 = p_1 t_2' + p_2 U_2 \qquad W_2 = \frac{V_2}{t_2}$$

$$V_1 = p_1 t_1' + p_2 U_1 \qquad W_1 = \frac{V_1}{t_1}$$

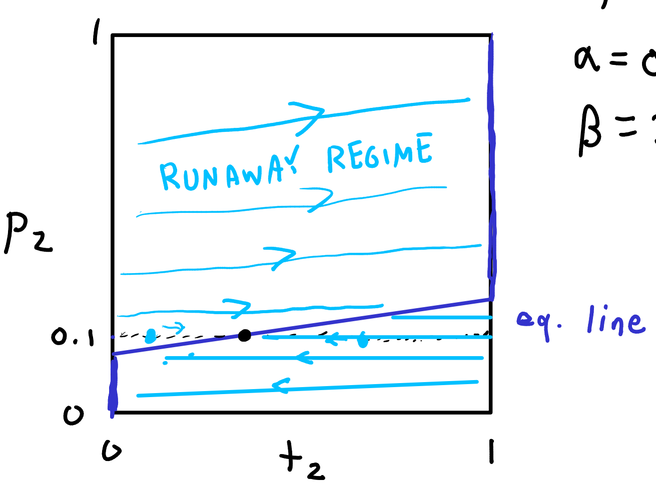
eq: $W_1 = W_2$ defines an equilibrium line (prop. of T_1 & T_2 males in next gen. is stable)

$W_2 > W_1$: prop. of T_2 males increase

$W_1 < W_2$: " " T_1 males increase

equilibrium condition: $p_2 = B(1 + C t_2)$

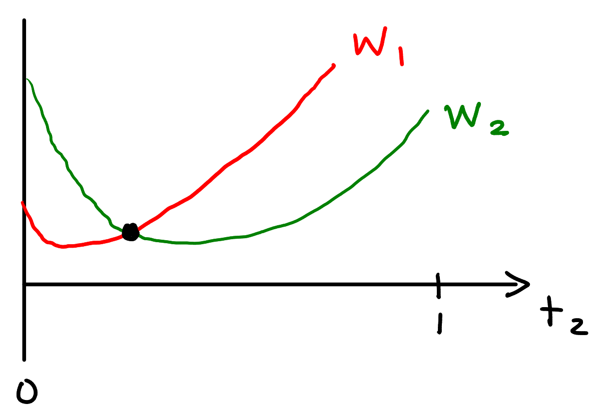
$$B = \frac{1-\alpha}{\alpha(\beta-1)} \qquad C = \alpha\beta - 1$$



$\alpha = 0.9$

$\beta = 3$

at $p_2 = 0.1$



runaway regime: ornament comes at a fitness cost but $t_2 \to 1$

Kin selection (inclusive fitness)

W. D. Hamilton [1964]: Hamilton's rule

an altruistic behavior (costly to the actor, but beneficial to others) would be favored by natural selection:

$$r \, b > c$$

r: degree of genetic relationship b/t actor + beneficiary

b: benefit to recipient

c: cost to the actor

mathematical framework: Price [Nature, 1970]

Price equation