Benefits of recombination: Hill-Robertson effect

$\Rightarrow$ (even just a little) recombination helps adaptive mutations take over quicker

simple example: two genes, two variants each

a or A , b or B

four types:

i		fitness f_i	$s_i = \frac{f_i}{f_1} - 1$
$i=1$	a b	1	0
2	A b	1.03	.03
3	a B	1.03	.03
4	A B	1.05	.05

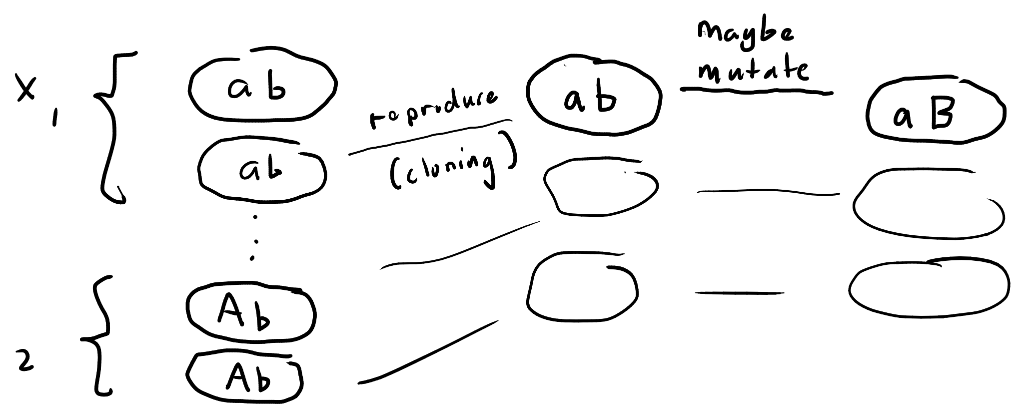
start w/ total pop. $N = 1000$ $\qquad Ns \gg 1$

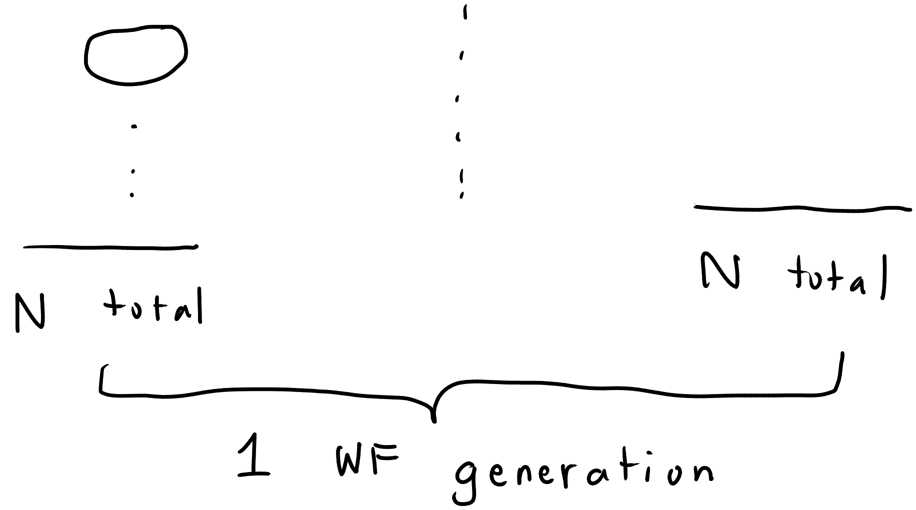
these are strongly adaptive mutants

fraction of type i; $x_i = \frac{n_i}{N}$

at $t=0$: $\vec{x} = (x_1, x_2, x_3, x_4)$
$= (1, 0, 0, 0)$ $\qquad$ 100% ab

initial simulation: Wright-Fisher w/ no recomb.





Cloning step: choose N new organisms from prev. generation, each type has prob:

$$p_i = \frac{f_i x_i}{\sum_j f_j x_j}$$

Mutation step: each gene has a prob. μ to get an adaptive mutation

$$a \xrightarrow{\mu} A$$

$$b \xrightarrow{\mu} B$$

(no back mutations)

$$\begin{tikzcd}
 & Ab \arrow[dr, "\mu"] & \\
ab \arrow[ur, "\mu(1-\mu)"] \arrow[rr, "\mu^2"'] \arrow[dr, "\mu(1-\mu)"'] & & AB \\
 & aB \arrow[ur, "\mu"'] &
\end{tikzcd}$$

μ small $\approx 10^{-4}$

$ab \to AB$ path unlikely compared

$ab \to Ab \to AB$

etc.

$\Rightarrow$ longer path to AB

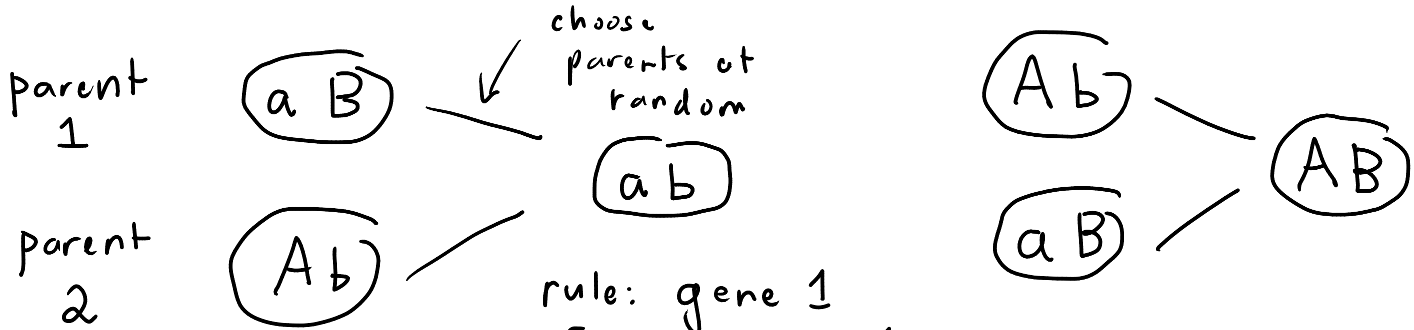
Simulation #2: add recomb.

$N - n$ organisms clone

n organisms recombine

fract. recomb.

$r = \frac{n}{N}$



rule: gene 1 from parent 1,
" 2 " 2

$i + j$ recombine w/ prob. $p_i p_j$