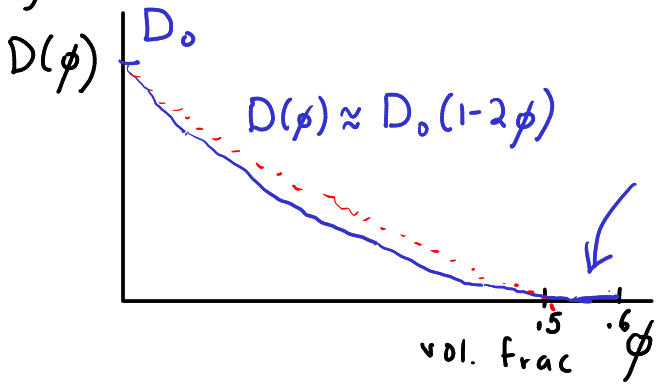
chemical reaction rate (3D diffusion)

$$k \leq k_{smol} = 4\pi D R c$$

D: $D_1 + D_2$; R: $R_1 + R_2$; c: $\frac{\#\ \text{searchers}}{\text{volume}}$

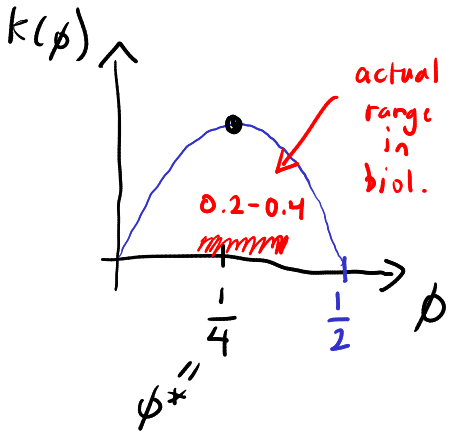


ϕ = fract. of cell volume occupied by proteins, DNA/RNA, etc.

$$k(\phi) = 4\pi D(\phi) R\, c(\phi)$$

$$\approx 4\pi D_0 (1-2\phi) R \frac{\phi}{V_{prot}}$$

$\approx$ dominated by proteins



$$\frac{dk}{d\phi} = 0 \Rightarrow \phi = \frac{1}{4}$$

$$\phi \cong \frac{n V_{prot}}{V}$$

n = # proteins

V_{prot} = typical prot. vol.

V: total cell volume

$$\Rightarrow \phi \cong c V_{prot}$$

$$c(\phi) = \frac{\phi}{V_{prot}}$$

$V_{prot} \approx 60\ nm^3$

typical $c \sim \frac{\phi^*}{V_{prot}} = \frac{1/4}{V_{prot}} = 7\ mM$

M = molar (M) $\approx \frac{N_A \rightarrow 6.022 \times 10^{23}}{L \rightarrow \text{liter}}$

cell volume $\sim 1\ \mu m^3$ for bacteria

$$\Rightarrow n = cV \sim 4 \times 10^6 \text{ proteins/cell}$$

actual E. coli values: $3 \times 10^6 - 10^7$ proteins/cell

optimal $k \approx 2.6 \times 10^6 \ s^{-1}$

timescales for reactions $\tau \sim 0.4 \ \mu s$

broad lessons:

any one protein $c < mM$, $n < 10^6$ } for $V \sim 1 \ \mu m^3$

more general $\phi = \dfrac{n_{prot} V_{prot} + n_{bp} V_{bp}}{V}$

$\sim .005$ for E. coli

bp = base pair of DNA/RNA

V_{bp} = vol. per bp