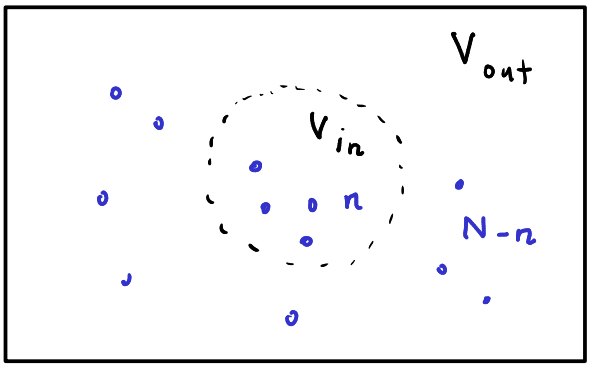
Transport across membranes:

I. Uncharged molecules

$c_{out} = \frac{N-n}{V_{out}}$

$c_{in} = \frac{n}{V_{in}}$

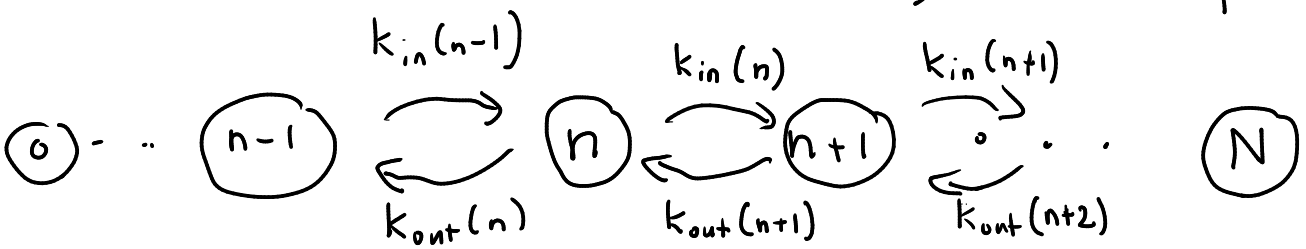


$V_{tot} = V_{in} + V_{out}$

for simplicity: V_{out}, V_{in} fixed

model: n part. inside

state n $\Rightarrow$ N-n part. outside



LDB condition:

add a part to outside

remove a particle from inside

$$\frac{k_{out}(n)}{k_{in}(n-1)} = e^{-\beta\left(-\mu_{in}(n) + \mu_{out}(n)\right)}$$

$$\mu_{out}(n) = \mu_0 + k_B T \ln c_{out}(n)$$

$$\mu_{in}(n) = \mu_0 + k_B T \ln c_{in}(n)$$

$$\frac{k_{out}(n)}{k_{in}(n-1)} = \frac{c_{in}(n)}{c_{out}(n)}$$

generically $n >> 1$

$$\frac{k_{out}(n)}{k_{in}(n)} \simeq \frac{c_{in}(n)}{c_{out}(n)}$$

net rate of part. leaving V_{in}:

$$F(n) = k_{out}(n) - k_{in}(n)$$

$$= k_{out}(n)\left(1 - \frac{k_{in}(n)}{k_{out}(n)}\right)$$

$$= k_{out}(n)\left(1 - \frac{C_{out}(n)}{C_{in}(n)}\right)$$

$$= \frac{k_{out}(n)}{C_{in}(n)}\left(C_{in}(n) - C_{out}(n)\right)$$

there is an equil. state n_0 where

$$C_{in}(n_0) = C_{out}(n_0) \Rightarrow F(n_0) = 0$$

$$\frac{n_0}{V_{in}} = \frac{N - n_0}{V_{out}} \Rightarrow n_0 = N\frac{V_{in}}{V_{in}+V_{out}}$$

$$= N\frac{V_{in}}{V_{tot}}$$

Taylor expand $F(n)$ around $n = n_0$:

$$\frac{k_{out}(n)}{C_{in}(n)} \approx \frac{k_{out}(n_0)}{C_{in}(n_0)} \equiv \frac{k_0}{C_0}$$

$$+ O(n - n_0)$$

$$C_{in}(n) - C_{out}(n) \approx O(n - n_0)$$

$$F(n) \approx \frac{k_0}{c_0}\left(c_{in}(n) - c_{out}(n)\right)$$

net flux of particles per unit area

$$\frac{F(n)}{A} = \underbrace{\frac{k_0}{c_0 A}}_{\text{permeability } \rho}\left(c_{in} - c_{out}\right)$$

$$\rho = \frac{k_0}{c_0 A} = \frac{s^{-1}}{nm^{-3}\, nm^2} = \frac{nm}{s}$$

rough estimates: $V_{out} \to \infty$, $c_{out} \to 0$

$n_0 \to 0$

mean # of particles $\langle n \rangle$

$$\frac{d\langle n\rangle}{dt} \approx -F(n) = \rho A\, c_{in}$$

divide by V_{in}

$$\frac{dc_{in}}{dt} \approx -\underbrace{\frac{\rho A}{V_{in}}}_{\equiv 1/\tau} c_{in}$$

$$\tau = \frac{V_{in}}{\rho A}$$

$$\Rightarrow c_{in}(t) = c_{in}(0)\, e^{-t/\tau}$$

Na^+: $\rho \sim 10^{-5}$ nm/s $R = 1000$ nm

$$\Rightarrow \tau \sim 3 \times 10^7 \text{ s} > 1 \text{ yr}$$

essentially impermeable

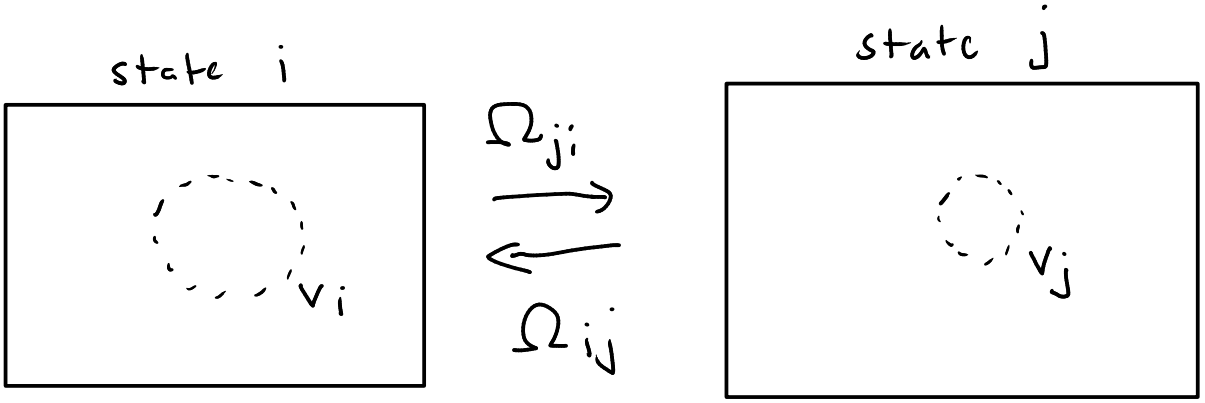
H_2O: $\rho \sim 10^5$ nm/s

$\tau \sim 3 \times 10^{-3}$ s very permeable

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takeaway: certain molec. are "stuck" in cell (unless you have other transport mechanisms)

model: a cell w/ a certain # of molec. that are "stuck" + the cell volume can change (b/c H_2O flows in/out)



n_α = # molec. of type α inside cell

$N_\alpha - n_\alpha$ = " " outside cell

$\alpha = 1, 2, \ldots, \#$ diff. types

$$\frac{\Omega_{ji}}{\Omega_{ij}} = e^{-\beta\left(-k_B T \ln \frac{\sigma_j}{\sigma_i}\right)}$$

σ_j = # microst. in j

σ_i = # microst. in i

focus on impermeable molec. (n_α doesn't change)

dilute approx.

$$\sigma_i = \binom{N_{pos,\alpha}^{in,i}}{n_\alpha}\binom{N_{pos,\alpha}^{out,i}}{N_\alpha - n_\alpha}\underbrace{(\quad)(\quad)\ldots}_{\text{pair for every type}}$$

$$N_{pos,\alpha}^{in,i} = \frac{V_i}{V_{mol,\alpha}} \qquad N_{pos,\alpha}^{out,i} = \frac{V_{tot} - V_i}{V_{mol,\alpha}}$$

↑ vol. of type α molec.

$$\ln \binom{M}{m} \approx -m \ln \frac{m}{M} \qquad m \gg 1,\ M \gg 1,\ M \gg m$$

$$\Rightarrow \ln \sigma_i \approx -\sum_\alpha \left[n_\alpha \ln \frac{n_\alpha V_{mol,\alpha}}{V_i} + (N_\alpha - n_\alpha) \ln \frac{(N_\alpha - n_\alpha) V_{mol,\alpha}}{V_{tot} - V_i} \right]$$

recall: $\frac{\Omega_{ji}}{\Omega_{ij}} \approx e^{-\beta(E_j - E_i)} \xrightarrow[t\to\infty]{} p_i^s = \frac{e^{-\beta E_i}}{Z}$

here: $\frac{\Omega_{ji}}{\Omega_{ij}} = e^{-\beta(-k_B T \ln \sigma_j - (-k_B T \ln \sigma_i))}$

$$\xrightarrow[t\to\infty]{} p_i^s = \frac{e^{-\beta(-k_B T \ln \sigma_i)}}{Z}$$

$$= \frac{e^{\ln \sigma_i}}{Z}$$

most likely state: highest p_i^s

$\Rightarrow$ largest $\ln \Omega_i$

$$\frac{d}{dV_i} \ln \Omega_i = 0 \Rightarrow$$

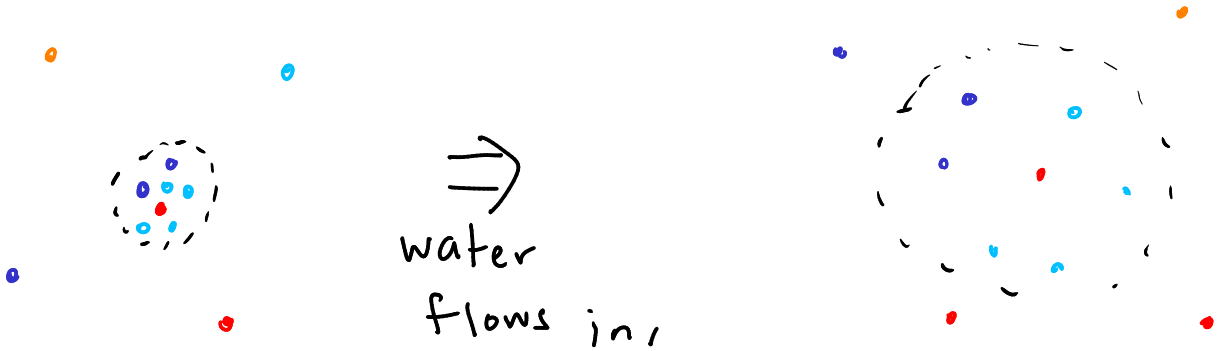
algebra

$$\sum_\alpha \left[-\underbrace{\frac{n_\alpha}{V_i}}_{C_\alpha^{in}} + \underbrace{\frac{(N_\alpha - n_\alpha)}{V_{tot} - V_i}}_{C_\alpha^{out}} \right] = 0$$

C_α^{in}: type α inner conc.

C_α^{out}: type α outer conc.

most likely state has: $\sum_\alpha C_\alpha^{in} = \sum_\alpha C_\alpha^{out}$



$\Rightarrow$ water flows in, sys. equil.

conc. mismatch $\Rightarrow$ volume changes

$\Rightarrow$ "osmotic stress"