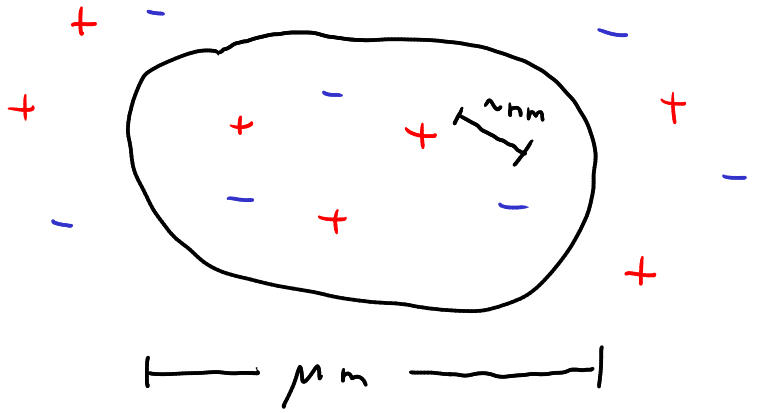
II. Charged molecules & transport across membranes

starting point: many charged molecules in biological fluids but there is an approx. balance betw. + and − charges ⇒ electroneutrality



typical charge concentrations: a few nm betw. charges

pair of +e and −e ions:

Coulomb energy of attraction: $= \frac{-e^2}{4\pi\varepsilon r}$

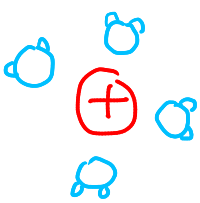
r = separation ~ 1 nm

ε = permittivity of surrounding material

$= \varepsilon_r \varepsilon_0$

ε_0 → vacuum perm.

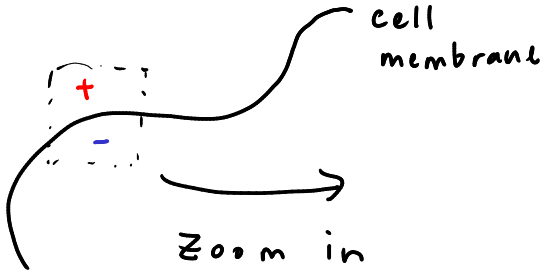
ε_r → relative perm: $\begin{cases} \approx 1 & \text{air} \\ \approx 80 & \text{water} \end{cases}$

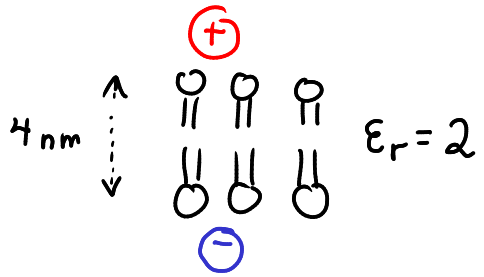


water molec. reorientation weakens overall field seen by ⊖

$r = 1$ nm, $\varepsilon_r = 80 \Rightarrow$ energy $\sim -0.7\ k_B T$

$\Rightarrow$ weak attraction easily broken by thermal fluctuations





$\Rightarrow$ Coulomb energy $= -8\ k_B T$

quite strong, stable against thermal fluctuations

if you have extra charges $\Rightarrow$ build up on either side of membrane

$\Rightarrow$ system that locally resembles a parallel plate capacitor

recall capacitor basics:

$$\rho = \frac{Q}{A} = \frac{\text{charge}}{\text{area}}$$

voltage drop $V = \frac{\rho d}{\varepsilon}$

d = separation

ε = permittivity inside

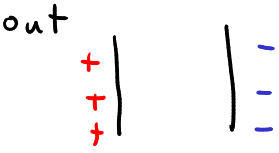
capacitance $C = \frac{Q}{V} = \frac{\varepsilon A}{d}$

specific capacitance $\frac{C}{A} = \frac{\varepsilon}{d} \sim 1\ \mu F/cm^2$

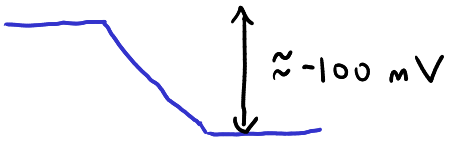
typical resting voltage across cell membranes

$V \approx -50$ to -200 mV

by convention measured from outside to in



potential:



$A \sim 3\ \mu m^2$

$\frac{Q}{e} = \frac{CV}{e} \approx 2 \times 10^4$ charges on surface

total ion concentrations: ~ 100 mM

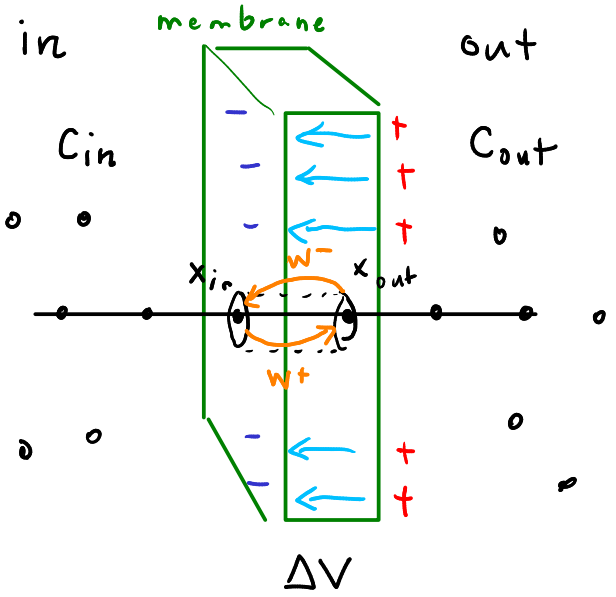
$\approx 10^8$ charges per fL

bact. cell $\approx$ fL

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simplest cross membrane ion transport:

passive pore (hole in the membrane)



detailed balance

$$\frac{w_+}{w_-} = e^{-\beta\left(H_{out} - H_{in} + k_B T \ln \frac{c_{out}}{c_{in}}\right)}$$

$$= e^{-\beta\left(-q\Delta V + k_B T \ln \frac{c_{out}}{c_{in}}\right)}$$

$$\Delta V = V_{in} - V_{out} = -100\ mV$$

$$|e\Delta V| \approx 4\, k_B T$$

When ( ) term $= 0 \Rightarrow w_+ = w_-$

no net movement of charge

$$\Rightarrow \quad -q\Delta V + k_B T \ln \frac{c_{out}}{c_{in}} = 0$$

$$\Rightarrow \quad \Delta V = \frac{k_B T}{q} \ln \frac{c_{out}}{c_{in}} \equiv V_N$$

Nernst potential

↓

specific to an ion type

(depends on q, c_{in}, c_{out})

rewrite LDB: $\frac{w_+}{w_-} = e^{-\beta q (V_N - \Delta V)}$

$q = +e$

if $\Delta V > V_N \Rightarrow w_+ > w_-$ net flow out

$\Delta V < V_N \Rightarrow w_- < w_+$ net flow in

imagine starting in scenario w/ $\Delta V > V_N$ q=te

$\Rightarrow$ flow out: leads to more excess $\oplus$ on outside

$\Rightarrow$ decrease in ΔV (c_{in}, $c_{out} \approx$ same)

$\Downarrow$

$V_N \approx$ same

$\Rightarrow$ eventually reach $\Delta V \approx V_N$

$\Rightarrow$ flow stops

if we start w/ $\Delta V < V_N \Rightarrow \Delta V$ increasing until flow stops

$\Rightarrow$ feedback mechanism that drives $\Delta V \rightarrow V_N$