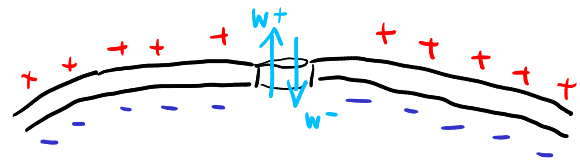
out K^+ $c_{out} = 20\,mM$



$\updownarrow \Delta V = V_{in} - V_{out}$

in K^+ K^+ K^+ $c_{in} = 400\,mM$ $q = +e$

$$\frac{W^+}{W^-} = e^{-\beta q (V_N - \Delta V)} \qquad V_N = \frac{k_B T}{q} \ln \frac{c_{out}}{c_{in}} = -75\ mV$$

typical $\Delta V = -65\ mV$

here $\Delta V > V_N \Rightarrow W^+ > W^-$ net flow out

net current out of $I \propto W_+ - W_-$

Taylor expansion: $I = 6(\Delta V - V_N) + \cdots$

↳ membrane conductivity (can reflect multiple channels)

$$\frac{\text{current}}{\text{area}} = j = \frac{I}{A} = g(\Delta V - V_N)$$

$$g \equiv \frac{6}{A} \approx 10\ \Omega^{-1} m^{-2}$$

[for K^+ channels in giant axons of squid]

Generalize: multiple ion typer, each with own channel type

for type i: $j_i = g_i (\Delta V - V_N^{(i)})$

$$V_N^{(i)} = \frac{k_B T}{q_i} \ln \frac{c_{out}^{(i)}}{c_{in}^{(i)}}$$

most important: $i = Na^+, K^+$ $\quad q_i = +e$

	c_{in}	c_{out}	$V_N^{(i)}$
Na^+	50 mM	440 mM	54 mV
K^+	400 mM	20 mM	-75 mV

resting potential (zero current) for squid is $\Delta V = V_0 = -65$ mV (for any ion) — how?

$$j_{Na} = g_{Na} \underbrace{(V_0 - V_N^{Na})}_{<0} + \underbrace{j_{Na}^{pump}}_{>0} = 0$$

$$j_K = g_K \underbrace{(V_0 - V_N^{K})}_{>0} + \underbrace{j_K^{pump}}_{<0} = 0$$

missing ingredient: Na/K pump

$\Rightarrow$ takes 3 Na^+ out + 2 K^+ in for every cycle, consuming one ATP

neuron / kidney cells: 50-70% of total ATP consumption is due to these pumps

three ingredients:

we will add this later

Na channels (g_{Na} will be function of ΔV)

K channels (passive: g_K constant)

Na/K pumps

$$j_{Na} = g_{Na}(V_0 - V_N^{Na}) + j_{Na}^{pump} = 0 \quad (1)$$

$$j_K = g_K(V_0 - V_N^{K}) + j_K^{pump} = 0 \quad (2)$$

$$j_K^{pump} = -\frac{2}{3} j_{Na}^{pump} \quad (3)$$

$\Rightarrow$ solve for $V_0 = \dfrac{2 g_{Na} V_N^{Na} + 3 g_K V_N^K}{2 g_{Na} + 3 g_K}$

$V_N^{Na} = 54$ mV

$V_N^K = -75$ mV

normal resting state

$g_K >> g_{Na}$

$g_K = 25\, g_{Na}$ for squid

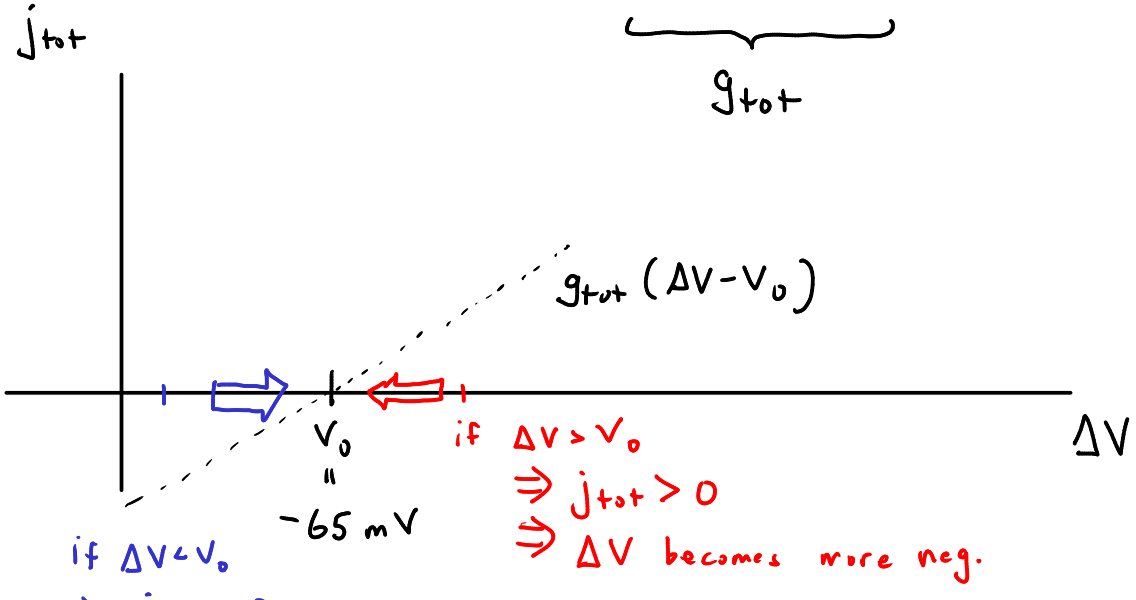
$\Rightarrow$ V_0 close to V_N^K

$V_0 = -65$ mV

What happens when $\Delta V \neq V_0$?

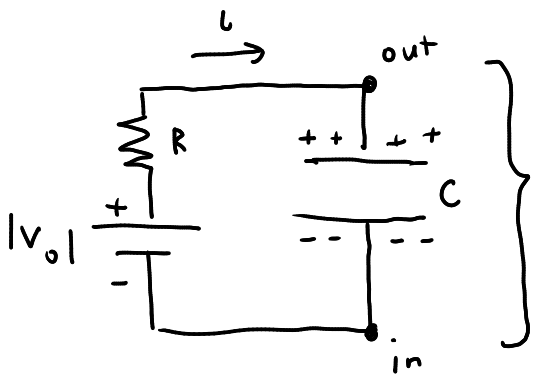
$$\Delta V = V_0 + u \qquad \text{where} \quad u \neq 0$$

$$j_{tot} = j_K + j_{Na} = g_K \left(u + V_0 - V_N^K \right) + j_K^{pump}$$
$$+ g_{Na} \left(u + V_0 - V_N^{Na} \right) + j_{Na}^{pump}$$
$$= \underbrace{\left(g_K + g_{Na} \right) u}_{g_{tot}} \qquad u = \Delta V - V_0$$



system is always pushed back to equil. at V_0

What is a circuit equivalent to this model? (ignoring spatial dep. of ΔV $\Rightarrow$ valid for small cells)



$$V = V_{in} - V_{out} = V_0 + u$$

$$R = \frac{1}{g_{tot} A}$$ → area of membrane

$$i = j_{tot} A = \underbrace{g_{tot} A}_{R^{-1}} (V - V_0) = -C \frac{dV}{dt}$$

minus sign is due to sign conv. for membranes