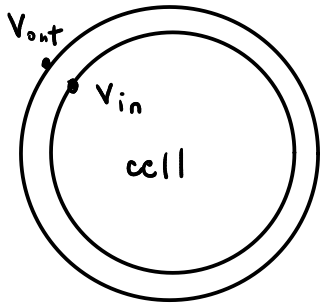
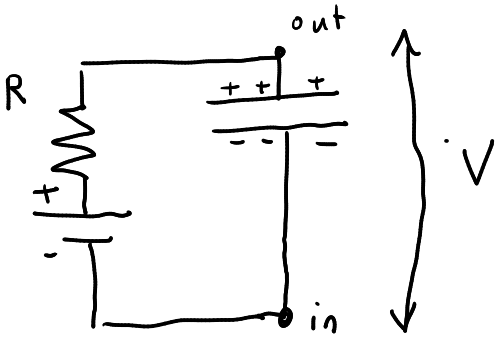


A_{mem} = memb. area $\Rightarrow$



$V = V_{in} - V_{out} \Rightarrow$ at rest $V = V_0 = -65$ mV

$R = \dfrac{1}{g_{tot} A_{mem}}$

$C_s = \dfrac{\text{capac.}}{\text{area}}$ $\quad C = C_s A_{mem}$

current thru membrane: $\quad \overbrace{R^{-1}}\ \overbrace{u}$

$$i = j_{tot} A_{mem} = g_{tot} A_{mem} (V - V_0)$$

RC circuit

$$= -C \frac{dV}{dt}$$

$$\Rightarrow \quad \frac{dV}{dt} = -\frac{1}{RC}(V - V_0)$$

solution: $V(t) = V_0 + e^{-t/RC}(V_{init} - V_0)$

V_{init} ↑ initial potential

$V(t) \to V_0$ as

$t >> RC$

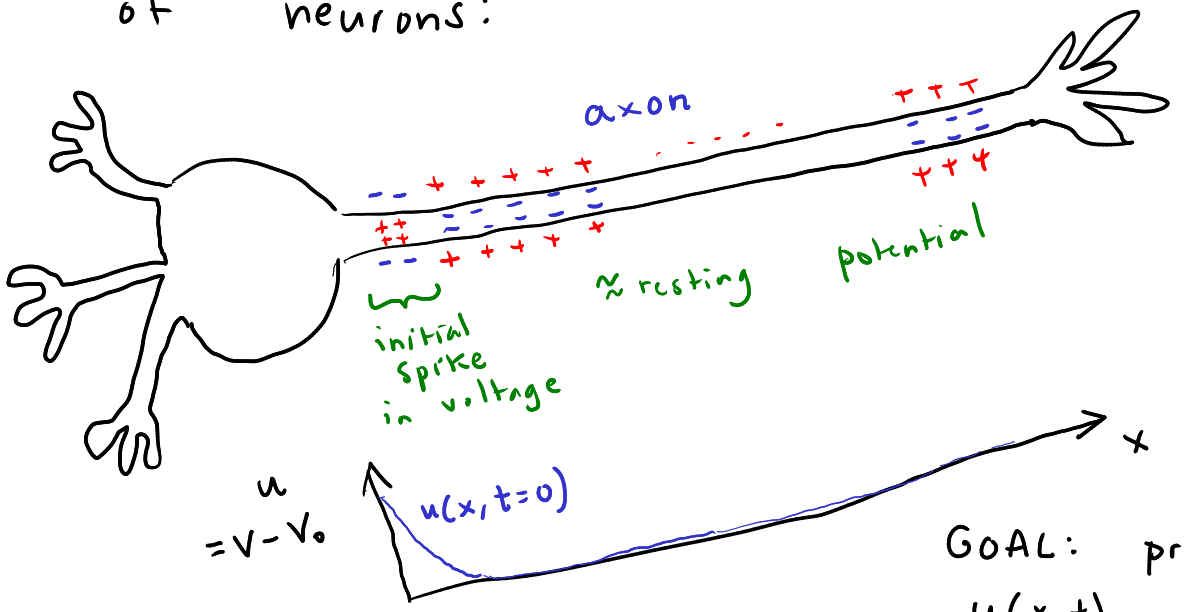
$$RC = \frac{C_s A_{mem}}{g_{tot} A_{mem}} = \frac{C_s}{g_{tot}}$$

$C_s \sim 1\ \mu F/m^2$

$g_{tot} \sim 10\ \Omega^{-1} m^{-2}$

~ 1 ms

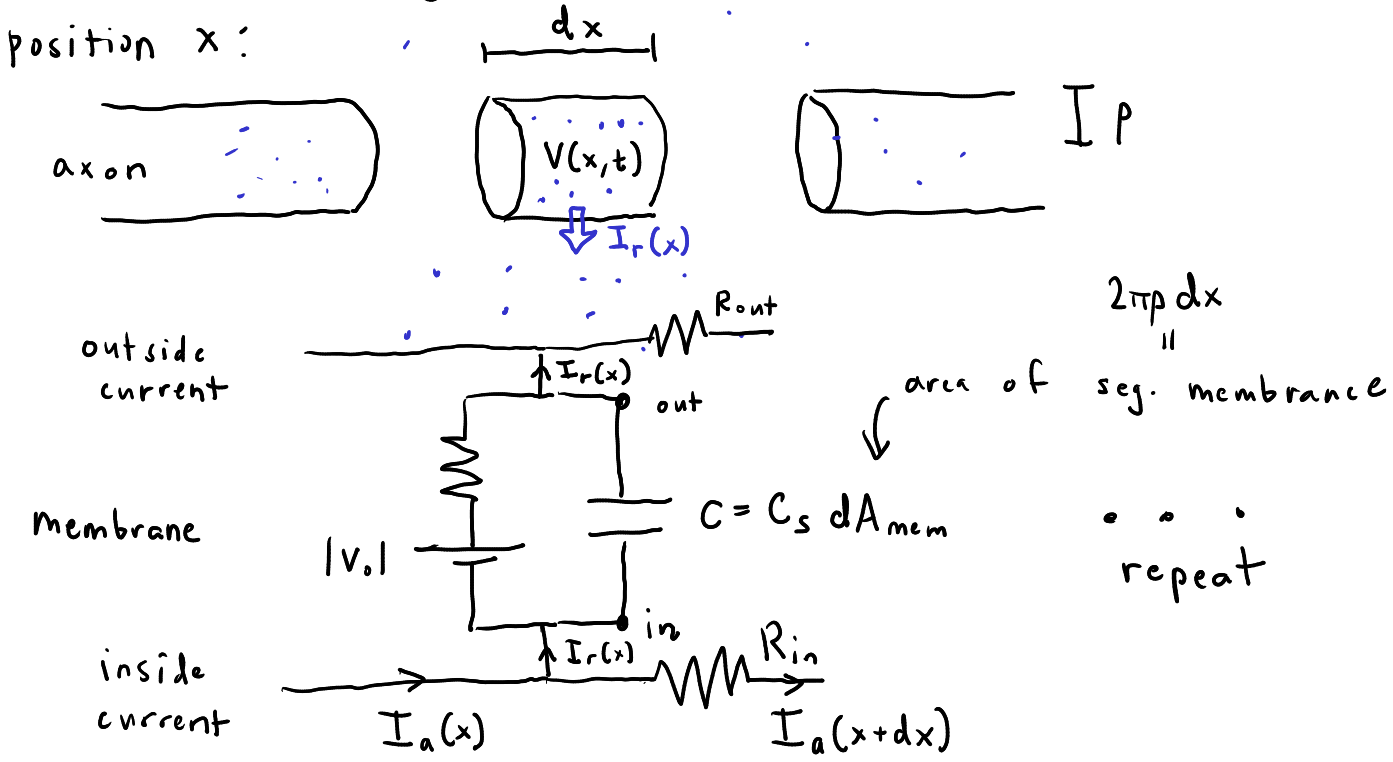
Next step: model long portions (axons) of neurons:



GOAL: predict $u(x,t)$ at later times ⇒ does the spike propagate down the axon

action potential

small slice of length dx at position x:



$2\pi\rho\, dx$ = area of seg. membrance

. . .

repeat

Pouilletis law: resistance of cylindrical wire

$$R = \frac{L}{\sigma A}$$



σ = conductivity of material

$$R_{in} = \frac{dx}{K \pi \rho^2}$$

K = conductivity of inside of axon

$\propto$ conc. of total ions

$\sim 3\ \Omega^{-1}\ m^{-1}$

$R_{out} \cong 0$

b/c A is huge for outside of axon

Question: how does spike propagate?

potential difference inside axon from one dx segment to next:

$$dV = -I_a(x) R_{in}$$

$$= -I_a(x) \frac{dx}{K\pi\rho^2}$$

$$\Rightarrow I_a(x) = -K\pi\rho^2 \frac{dV}{dx} \quad (1)$$

change in axial current from seg. to seg.

$$\underbrace{I_a(x) - I_a(x+dx)}_{-dx \frac{dI_a}{dx}} = I_r(x)$$

$$= dA_{mem}\, g_{tot} (V - V_0) + dA_{mem}\, C_s \frac{dV}{dt}$$

$$\Rightarrow -dx \frac{dI_a}{dx} = 2\pi\rho\, dx \left[g_{tot}(V - V_0) + C_s \frac{dV}{dt} \right] \quad (2)$$

plug (1) into (2), divide by dx:

$$K\pi\rho^2 \frac{d^2V}{dx^2} = 2\pi\rho \left[\underbrace{g_{tot}(V-V_0)}_{j_{tot}(V)} + C_s \frac{dV}{dt} \right]$$

$j_{tot}(V)$ = total ion curr. thru memb.

$$u(x,t) = V(x,t) - V_0 \qquad \lambda \equiv \sqrt{\frac{\rho K}{2 g_{tot}}} \quad [\text{units: length}]$$

$$\tau = \frac{C_s}{g_{tot}} \quad [\text{units: time}]$$

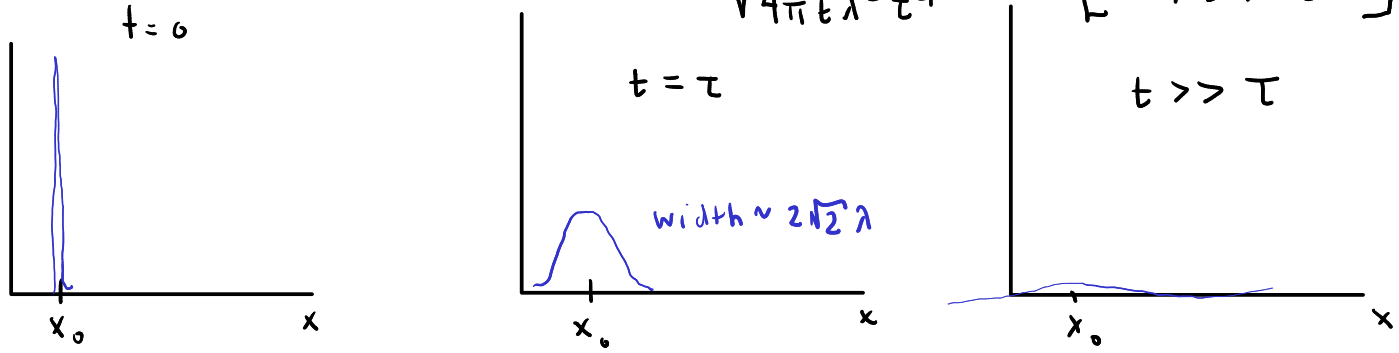
$$\Rightarrow \boxed{\lambda^2 \frac{\partial^2 u}{\partial x^2} - \tau \frac{\partial u}{\partial t} = u}$$

cable equation

trivial sol'n: $u(x,t) = 0$ everything at resting potential

initial spike: $u(x,0) = B\,\delta(x-x_0)$

$$u(x,t) = \frac{e^{-t/\tau}}{\sqrt{4\pi t \lambda^2 \tau^{-1}}} \exp\left[-\frac{(x-x_0)^2}{4 t \lambda^2 \tau^{-1}} \right]$$



Squid: $\rho = 0.5$ mm, $K = 3\ \Omega^{-1} m^{-1}$, $C_s = 1\ \mu F/cm^2$, $g_{tot} = 10\ \Omega^{-1} m^{-2}$

$\Rightarrow$ $\lambda = 9$ mm, $\tau = 1$ ms

speed of spread : $\frac{\text{width}}{\text{time}} \sim \frac{\lambda}{\tau} \sim 9$ m/s

spike gets smeared but does not move

$\Rightarrow$ need something else!