$u(x,t) = V(x,t) - V_0$

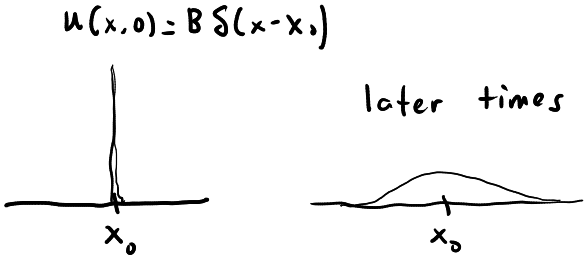


$$\lambda = \sqrt{\frac{\rho K}{2 g_{tot}}} \qquad \tau = \frac{C_s}{g_{tot}}$$

cable equation:

$$\lambda^2 \frac{\partial^2 u}{\partial x^2} - \tau \frac{\partial u}{\partial t} = f(u)$$

linear case: $f(u) = u$



$f(u)$ term comes from j_{tot} = total current thru membrane

term on RHS: $2\pi\rho \underbrace{g_{tot}(V-V_0)}_{j_{tot}(V)}$

for arbitrary $j_{tot}(V)$ function: $f(u) \equiv \dfrac{j_{tot}(V_0+u)}{g_{tot}}$

goal: can we find a more general, nonlinear version of $j_{tot}(V)$?

Recall: how we derived $j_{tot}(V) = g_{tot}(V-V_0)$

$\Rightarrow$ under assumption most Na channels were closed

$$V_0 = \frac{2 g_{Na} V_N^{Na} + 2 g_k V_N^K}{2 g_{Na} + 3 g_k}$$

$V_N^{Na} = 54$ mV

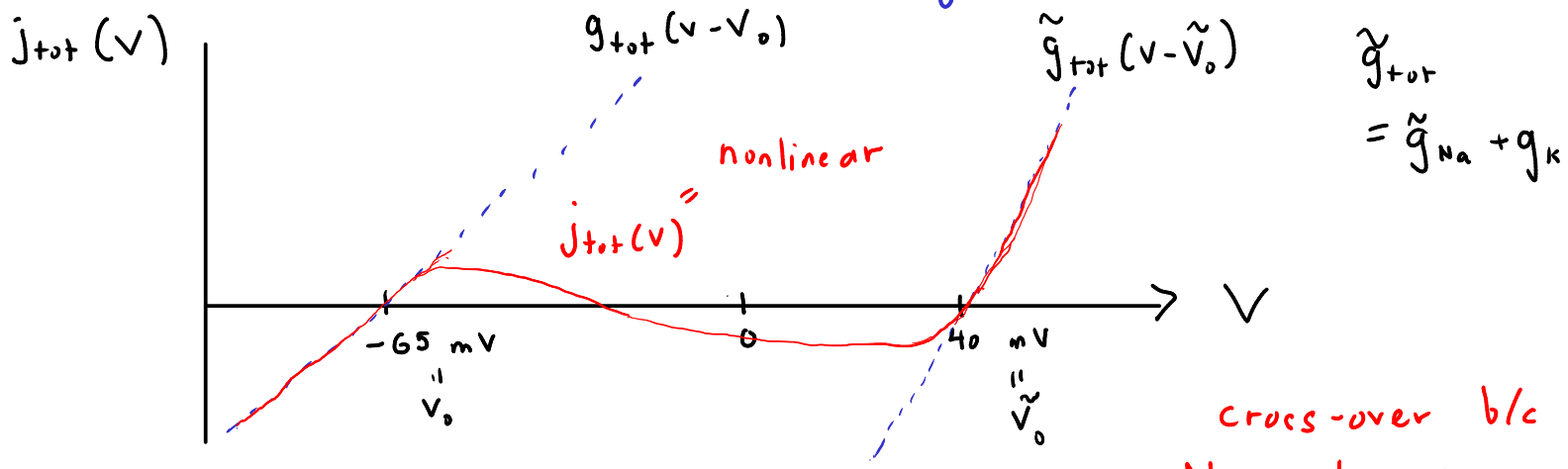
$V_N^K = -75$ mV

Scenario I: Na channels closed: $g_{Na} \ll g_K$

V_0 closer to V_N^K

$g_{tot} = g_{Na} + g_K$

$V_0 = -65$ mV



cross-over b/c Na channels open w/ increasing V

Scenario II: most Na channels open

new value $\tilde{g}_{Na} >> g_K$ (many more Na channels)

new $\tilde{V}_0 = 40$ mV closer to V_N^{Na}

Look for solutions of:

$$\lambda^2 \frac{\partial^2 u}{\partial x^2} - \tau \frac{\partial u}{\partial t} = f(u)$$ w/ nonlinear

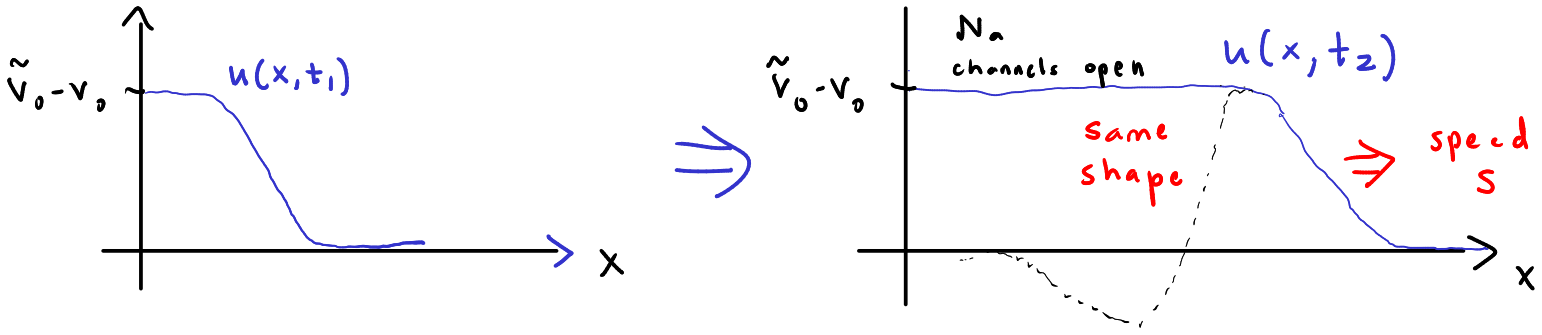
$$f(u) = \frac{j_{tot}(V_0 + u)}{g_{tot}}$$

↓

guess a traveling wave sol'n to this equation & prove it works

$\Rightarrow$ solution is called a soliton:

traveling wavefront that keeps its form b/c of a balance of dissipation (ions leaking thru memb.) + reinforcement (opening of Na channels)



assume:

$$u(x,t) = (\tilde{V}_0 - V_0)\, w\left(\frac{x - st}{\lambda}\right)$$

θ = dimensionless

λ: length

$\uparrow$ missing "depolarization" part

$\Rightarrow$ include this later

w: unknown shape function w/ following properties:

$$w(\theta \to -\infty) = 1$$

$t \to \infty \Rightarrow u = \tilde{V}_0 - V_0$ (Na channels open)

$$w(\theta \to +\infty) = 0$$

$t \to -\infty \Rightarrow u = 0$ (Na channels open)

plug into cable equation:

$$\frac{\partial^2 u}{\partial x^2} = (\tilde{V}_0 - V_0)\frac{1}{\lambda^2}\frac{\partial^2 w}{\partial \theta^2}$$

$$\frac{\partial u}{\partial t} = -(\tilde{V}_0 - V_0)\frac{s}{\lambda}\frac{\partial w}{\partial \theta}$$

$\Rightarrow$ write down cable eqn, div. by $\tilde{V}_0 - V_0$:

$$\frac{\partial^2 w}{\partial \theta^2} = -\frac{\tau s}{\lambda}\frac{\partial w}{\partial \theta} + \underbrace{F(w)}_{\equiv \frac{f(w)}{\tilde{V}_0 - V_0}}$$

$\Rightarrow$ try to solve this for $w(\theta)$