

$w(\theta \to -\infty) = 1$

$w(\theta \to +\infty) = 0$

$$u(x,t) = (\tilde{V}_0 - V_0)\, w\left(\underbrace{\frac{x-st}{\lambda}}_{\theta}\right)$$ guess

w: unknown wavefront shape

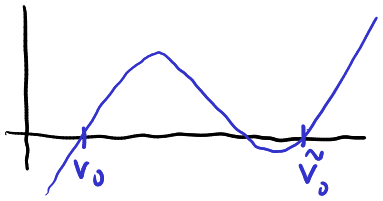
$$\lambda = \sqrt{\frac{\rho\kappa}{2g_{tot}}} \qquad \tau = \frac{c_s}{g_{tot}}$$

rewrote cable equ:

$$\frac{\partial^2 w}{\partial \theta^2} = -\underbrace{\frac{\tau s}{\lambda}}_{\gamma} \frac{\partial w}{\partial \theta} + \underbrace{F(w)}_{\text{nonlinear func.}}$$

$$\equiv \frac{f(w)}{\tilde{V}_0 - V_0}$$

$$f(u) = \frac{j_{tot}(V_0 + u)}{g_{tot}}$$



$$\frac{\partial^2 w}{\partial \theta^2} = -\gamma \frac{\partial w}{\partial \theta} + F(w)$$

analogy to classical mech:

$\theta \sim$ "time"

$w \sim$ "distance"

$\frac{\partial^2 w}{\partial \theta^2} \sim$ "acceleration"

$m \sim$ "mass" $= 1$

external

$F(w) \sim$ "force"

$-\gamma \frac{\partial w}{\partial \theta}$

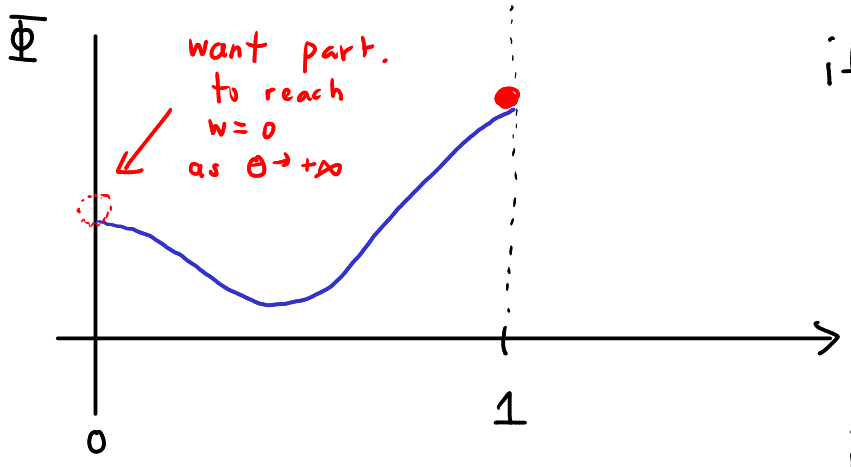
$\sim -$ drag coeff $\times$ velocity

$=$ drag friction

to solve: write $F(w)$ in terms of a potential energy

$$F(w) = -\Phi'(w)$$

$$\Rightarrow \Phi(w) = -\int_0^w dw' \, F(w') + \text{const.}$$



if $\gamma = 0$ (no frict.) part. would not stop at $w = 0$ as $\theta \to +\infty$

if γ too large then part. exhausts kinetic energy by the time it reaches valley $\Rightarrow$ no way to get to $w = 0$ as $\theta \to +\infty$

there exists a unique "Goldilocks" value $\gamma = \gamma_0$ where $w(\theta) \to 0$ as $\theta \to +\infty$

$$\Downarrow$$

$$\gamma = \frac{\tau s}{\lambda} = \gamma_0 \quad \Rightarrow \quad s = \underbrace{\frac{\lambda}{\tau}}_{\text{for a squid axon } \sim 9 \text{ m/s}} \gamma_0$$

γ_0 → some dimensionless number $\sim O(1)$

$$V_N = \frac{k_B T}{q} \ln \frac{c_{out}}{c_{in}}$$