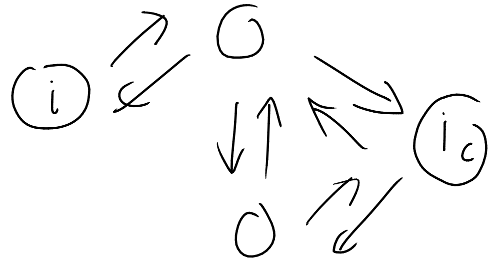
network: $\Omega_{ij}$




$\tau_i =$ mean (first) time from $i \to i_c$

$i \neq i_c$: $\displaystyle\sum_{j=1}^{N} \tau_j \Omega_{ji} = -1 \qquad \tau_{i_c} = 0$

recall motivation:


$r_c = R_1 + R_2$

diff. consts. $D_1, D_2$

sep. dynamics

diffuser w/ $D = D_1 + D_2$

start w/ 1D version: continuous space approx.

$$p_j(t) \longrightarrow p(x,t)$$

$$\frac{dp_i}{dt} = \sum_j \Omega_{ji} p_j \longrightarrow \frac{\partial p}{\partial t} = D \frac{\partial^2 p}{\partial x^2}$$

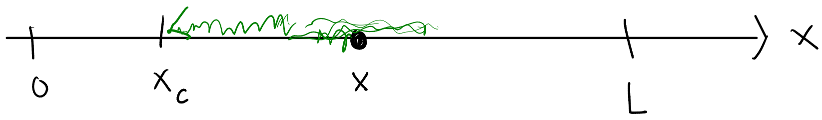
B.C. $\left.\dfrac{\partial p}{\partial x}\right|_{0,L} = 0$

same approach: $\tau_i \to \tau(x)$ = avg. time to go from $x$ to $x_c$

$$i_c \to x_c$$

$$i \neq i_c: \sum_j \tau_j \Omega_{ji} = -1 \rightsquigarrow D\frac{\partial^2 \tau}{\partial x^2} = -1$$

$$\tau(x_c) = 0 \qquad \left.\frac{\partial \tau}{\partial x}\right|_L = 0$$



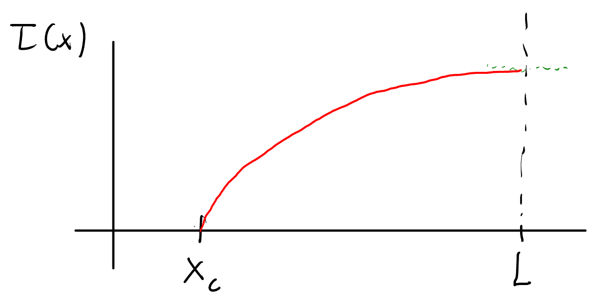

$$\frac{\partial^2 \tau}{\partial x^2} = -\frac{1}{D} \quad \Rightarrow \text{integrate both sides} \quad \frac{\partial \tau}{\partial x} = -\frac{1}{D}x + c_1$$

$$\Rightarrow \text{integrate again} \quad \tau(x) = -\frac{1}{2D}x^2 + c_1 x + c_2$$

B.C. $\left.\frac{\partial \tau}{\partial x}\right|_L = 0 = -\frac{1}{D}L + c_1 \Rightarrow c_1 = \frac{L}{D}$

$$\tau(x_c) = 0 \Rightarrow c_2 = \frac{x_c^2}{2D} - \frac{Lx_c}{D}$$

$\Rightarrow$ plug in consts: $\tau(x) = \dfrac{(x - x_c)(2L - x - x_c)}{2D}$




1D diffusion to capture

3D: $\tau(\vec{r})$ = mean time to go from sep. $\vec{r}$ to some sep. $\vec{r}_c$

where $r_c = |\vec{r}_c| = R_1 + R_2$

$= \tau(r)$ by symmetry (where $r = |\vec{r}|$)

$$D\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]\tau(\vec{r}) = -1$$


$$\tau(\vec{r} \in \text{sphere of radius } r_c) = 0$$

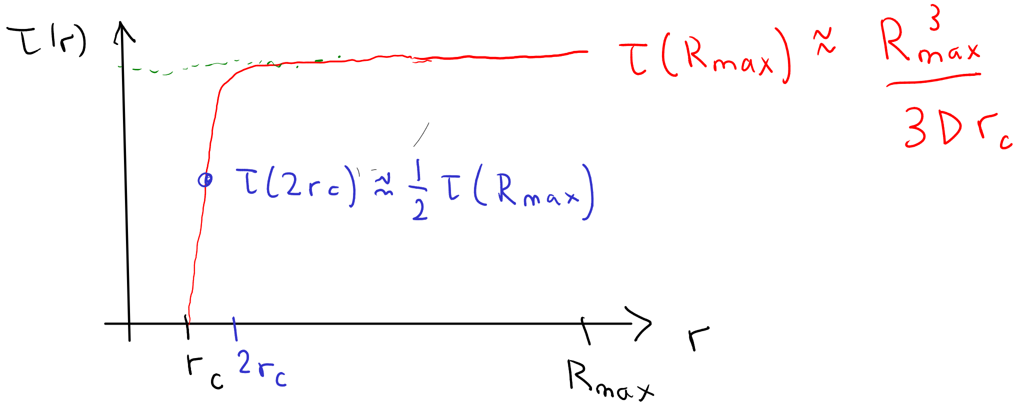
$$\nabla \tau(\vec{r})\Big|_{\vec{r} \in \text{outer boundary of max. sep.}} = 0$$

$$\vec{r} = (r, \theta, \phi) \qquad \tau(\vec{r}) = \tau(r)$$

$$\Rightarrow \quad D\left[\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r}\right]\tau(r) = -1$$

$$\tau(r_c) = 0 \qquad \frac{\partial \tau}{\partial r}\Big|_{R_{max}} = 0$$

$$\Rightarrow \quad \tau(r) = \frac{R_{max}^3}{3Dr_c} - \frac{R_{max}^3}{3Dr} - \frac{r^2}{6D} + \frac{r_c^2}{6D}$$




$\Rightarrow$ takeaway message: to a good approx (assuming dilute solution) time for two particles to meet in 3D

$$\tau \approx \frac{R_{max}^3}{3 D r_c}$$ (valid when initial sep. > few nm)

MSD $= L^2 = 6Dt$

$t = \frac{L^2}{6D}$

$$= \underbrace{\left(\frac{R_{max}^2}{6D}\right)}_{\text{avg. time for molec. sep. to cover a dist. } R_{max} \text{ via diffusion}} \underbrace{\left(\frac{2 R_{max}}{r_c}\right)}_{\text{additional factor related to how likely a collision occurs}}$$

typical protein $R_1 = R_2 \sim 1$ nm

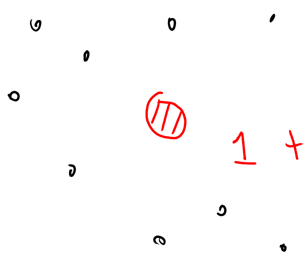
$R_{max} \sim 1$ $\mu$m for bacterium

$D \sim 10\ \mu m^2/s \Rightarrow \tau = (8.3 \times 10^{-3}\ s)(1000)$

$= 8.3$ s

in order to speed up chemical reactions $\Rightarrow$

have more particles



n "searcher" particles

1 target

$$\tau = \underbrace{\left(\frac{R_{max}^2}{6D}\right)}_{\text{time to explore whole cell}} \left(\frac{2 R_{max}}{r_c\, n}\right)$$

chance of collision increases by a factor of n (dec. time)

$R = r_c = R_1 + R_2$

$V = \frac{4}{3}\pi R_{max}^3$ (sph. cell volume)

$$\tau = \frac{V}{4\pi D R n} = \frac{1}{4\pi D R c}$$

$c = \frac{n}{V}$ = concentration of searcher particles rate of searchers hitting target

$$\boxed{k_{smol} = \frac{1}{\tau} = 4\pi D R c}$$

Smoluchowski rate

"speed limit" for any chem. reaction that requires only 3D diffusion