Smoluchowski rate: $k = 4\pi D R c$

- k: rate at which searchers find target
- D: $D_1 + D_2$
- R: $R_1 + R_2$
- c: conc. of searcher particles

any reaction rate $K \leq k_{smol} = 4\pi D R c$

How can biology optimize speed of chemistry?

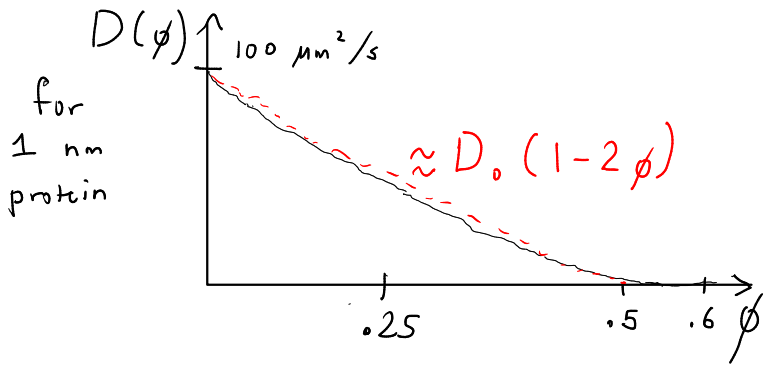
$\Rightarrow$ increasing c

$\Rightarrow$ cell more crowded

$\Rightarrow$ smaller diffusion const. D

ϕ = volume fraction of (non-water) stuff in cell (proteins, DNA/RNA, etc.)

natural range of ϕ: 0.2 – 0.4 in modern cells



ϕ = dominated by proteins

$\approx \frac{n V_{prot}}{V}$ ← total cell volume

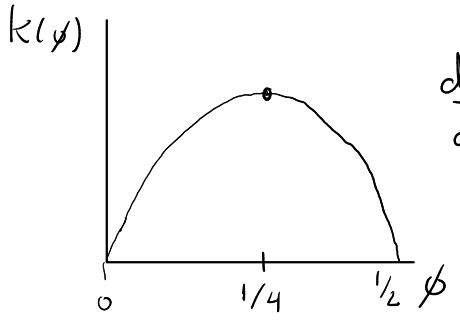
n = # proteins

$V_{prot} \approx$ Vol. per protein

$c(\phi) = \frac{\phi}{V_{prot}}$ $\Leftarrow$ $\phi = c V_{prot}$ $c =$ conc. of proteins

$$\max\ k(\phi) = 4\pi D(\phi) R\, c(\phi)$$

$$\approx 4\pi D_0 (1-2\phi) R \phi V_{prot}^{-1}$$



$$\frac{dk}{d\phi} = 0 \Rightarrow \phi = \frac{1}{4} \text{ at } \max k(\phi)$$

estimate total conc. of proteins in a cell:

$$c \sim \frac{\phi}{V_{prot}} = \frac{1/4}{60\,nm^3}$$ ← typical prot. vol.

conc. unit = molar (M) = $\frac{N_A}{L}$ → 6.022×10^{23}; L → liter

typical total conc. of proteins ~ $c = 7$ mM

cell volume of bacteria ~ $1\ \mu m^3$

$\Rightarrow n = cV \sim 4\times10^6$ proteins/cell

actual E. coli values: $3\times10^6 - 10^7$ proteins/cell

optimal $k \approx 2.6\times10^6\ s^{-1}$

reaction timescales $\tau \sim 0.4\ \mu s$

broad lessons: any one protein $c < mM$

$n < 10^6$ (for $V \sim 1\ \mu m^3$)

one last point: $\phi = \frac{n_{prot} v_{prot} + n_{bp} v_{bp}}{V}$

$\sim .005$ for E. coli

n_{bp} = # of base pairs of genetic material

V_{bp} = volume per bp