What types of W matrices are seen in physical systems?

$\Rightarrow$ In many cases W is micro. reversible and shows convergence $W^n \vec{p}^{\,0} = \vec{p}^{\,s}$ as $n \to \infty$

Need: physical framework

$\Rightarrow$ classical mechanics in d-dim. w/ M particles

$\left.\begin{array}{l} \vec{q} = dM \text{ coord. of all particles} \\ \vec{p} = dM \text{ momenta " " "} \end{array}\right\}$ 2dM dim. phase space

state of system $\Longleftrightarrow$ point $(\vec{q}, \vec{p})$ in this phase space

system has finite volume V, no gain/loss of energy from outside

total energy (conserved) $E = \mathcal{H}(\vec{q}, \vec{p})$ Hamiltonian

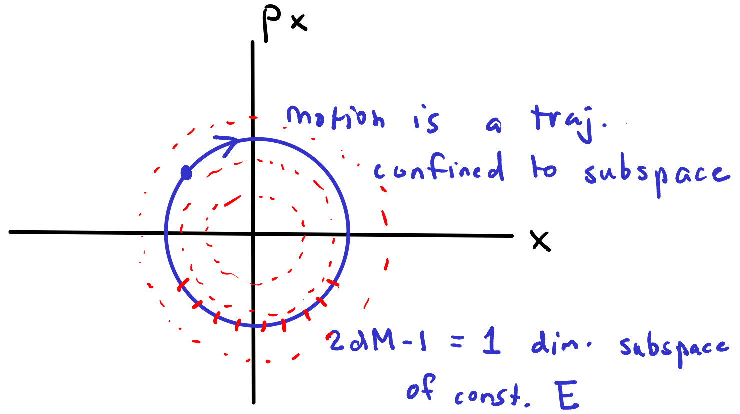
defines a subspace of const. E in a phase space w/ dim. $2dM - 1 \equiv D$

1D harm. oscillator

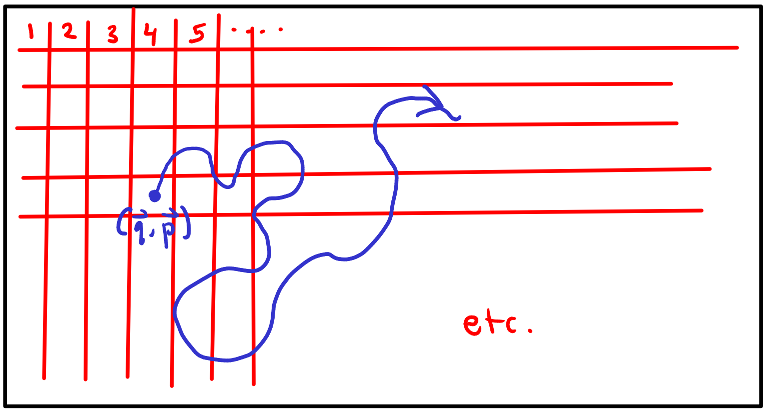
$\mathcal{H} = \frac{1}{2} p_x^2 + \frac{1}{2} x^2 = E$

phase space: $(x, p_x) \Rightarrow 2dM = 2$ dim. phase

$d = 1, \quad M = 1$



cartoon representation of a single "onion" layer = D-dim subspace



divide up subspace into D-dim. regions of volume a

label boxes $\mu = 1, 2, 3, \ldots\ldots, \Theta_a(E)$

↳ leave out for simplicity of notation (a arbitrary, but fixed)

box ≡ "microstate"

Stat. mech. focuses on systems w/ certain properties:

- ergodicity: for any trajectory, as $t \to \infty$ every microstate μ is visited (no matter how small a is)

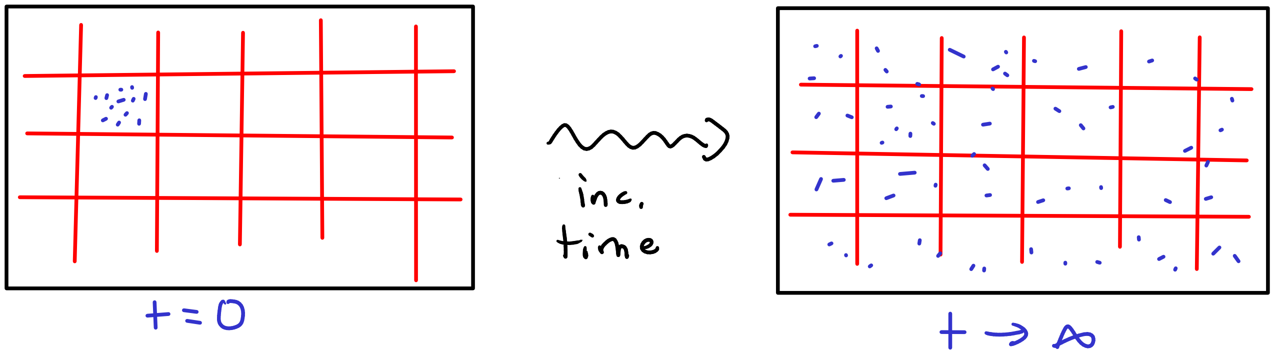
  note: in systems w/ nontrivial conserved quantities besides E this may be violated

  in 2d harm. oscillator $E = E_x + E_y$

$$E_x = \tfrac{1}{2}p_x^2 + \tfrac{1}{2}x^2$$
$$E_y = \tfrac{1}{2}p_y^2 + \tfrac{1}{2}y^2$$
} separately conserved

- mixing: if we initiate a group of trajectories at $t=0$ in same microstate μ

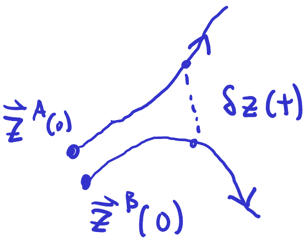
  $\Rightarrow$ traj. spread out evenly to all microstates



$$\text{ergodicity} \subset \text{mixing}$$

How is this possible?

Two traj. w/ diff initial conditions:



$$\vec{z} = (\vec{q}, \vec{p})$$

$$\vec{z}^A(t) \qquad \vec{z}^B(t)$$

$$\delta\vec{z}(t) = \vec{z}^A(t) - \vec{z}^B(t)$$

chaos: $\quad |\delta\vec{z}(t)| \sim e^{\lambda t} |\delta\vec{z}(0)|$ for initial times

w/ $\lambda > 0$

Lyapunov exponent

Plausibly this chaotic property (at short times) could lead to mixing (at long times)

$\Rightarrow$ but rigorous is quite hard!