Donut-centric view of universe

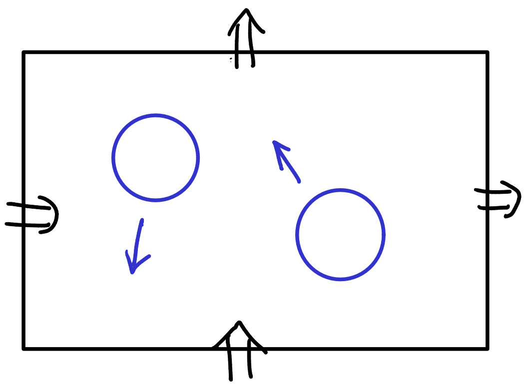
behavior of traj.		classical version	quantization
confined to donuts	class. mech.	integrable $\mathcal{H}(\vec{\phi},\vec{I}) = \mathcal{H}(\vec{I})$	$I_k = \hbar(n_k + c_k)$ ↓ integer ≥ 0 ↓ const. (Maslov correction)
confined to deformed donuts	class. mech.	near-integrable $\mathcal{H}(\vec{\phi},\vec{I}) = \mathcal{H}_0(\vec{I}) + \alpha U(\vec{\phi},\vec{I})$, α small	
not confined (no donuts!) chaos	stat. mech.	more general Hamiltonians	no simple rules

1970: first rigorous that a system could be ergodic + mixing $\Rightarrow$ Yakov Sinai

Sinai billiard

hard disks colliding

phase space: positions + velocities of disks



2d box w/ periodic B.C.

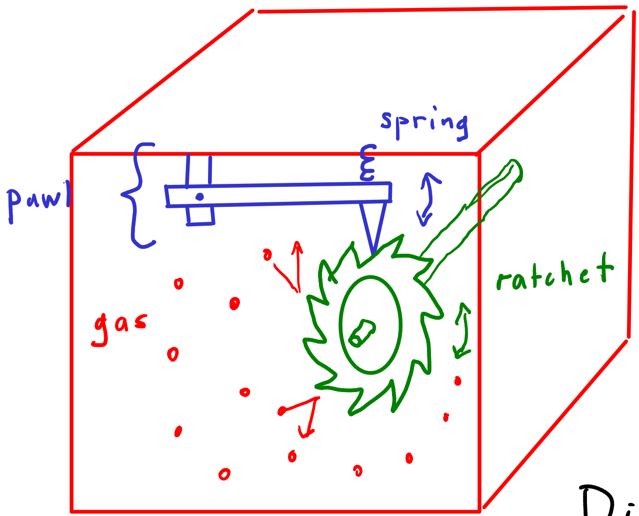
Rule of thumb:

if a system has many particles (deg. of freedom) + if they are "strongly" interacting $\Rightarrow$ assume ergodicity + mixing

$$\Rightarrow \quad P_\mu(t) \xrightarrow[t\to\infty]{} P_\mu^s = \frac{1}{\Theta(E)}$$

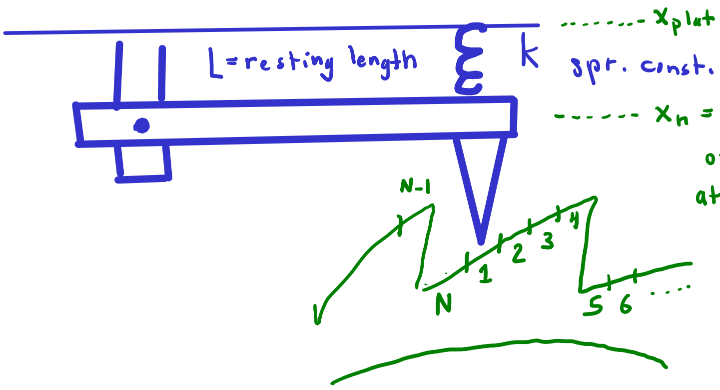
current state-of-art: $N \geq 2$ d-dim. spheres are almost proven ergodic

Illustrative example from Feynman: ratchet + pawl



- total system is isolated (total energy is conserved)
- assume total system is ergodic + mixing

Divide up into:



total = system + environment
(ratchet + pawl) (gas in box)

$$E_{tot} = E_{sys} + E_{env}$$

E_{sys} in nth state

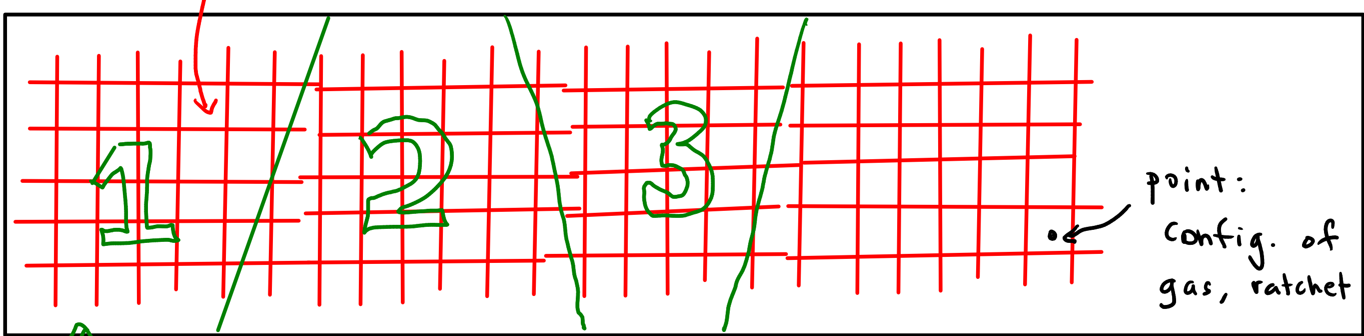
$$\equiv E_n = \frac{1}{2}k\left(x_{plat} - x_n - L\right)^2$$

$$E_{tot} = E_n + E_n^{env}$$

const.

↳ energy of env. when system is in state n

box: "microstate": group of similar configs.



point: config. of gas, ratchet + pawl

each "country" ≡ macrostate: all configs of total where system is in state n

entire phase for const tot. energy E_{tot}