box: "microstate": μ
group of similar configs.

point: config. of gas, ratchet + pawl

each "country" $\equiv$ macrostate: all configs of total where system is in state n

entire phase for const tot. energy E_{tot}

total is ergodic + mixing:

$$P_\mu(t) \xrightarrow[t\to\infty]{} \frac{1}{\Theta(E_{tot})} \equiv \frac{1}{\Theta_{tot}}$$

prob. to be in microstate μ (small box)

each μ has the same volume of phase space a by construction

Θ_n = # of microstates in macrostate n

$\Rightarrow$ could be diff. between diff. n

b/c E_n^{env} env. energy varies w/ n

assumptions:

$$\Theta_{tot} = \sum_{i=1}^{N} \Theta_n$$

tot # of microstates

N = # of macrostates

- envir. has many more degrees of freedom than sys:

$$\Theta_n \gg N$$

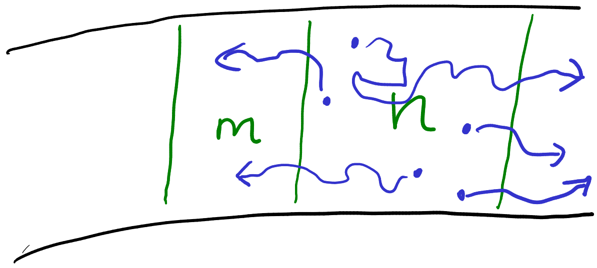
$$\Theta_{tot} \gg N$$

- mixing is fast
- dynamics of exploring within a macrostate happens faster than

transitions b/t macrostates

$\Rightarrow$ prob. of next macrostate visited depends at most on the current one (lose "memory" of deeper past b/c chaotic mixing dynamics)

transition matrix W_{mn} = prob. to go from macrostate $n \to m$ in time interval δt, given we are currently in n



$$= \frac{\#\text{ traj. that in start in } n \text{ + end up in } m \text{ after time } \delta t}{\#\text{ traj. '' '' '' in } n \text{ + end up anywhere after time } \delta t}$$

$P_n(t_i)$ = prob. of macrostate n at time t_i

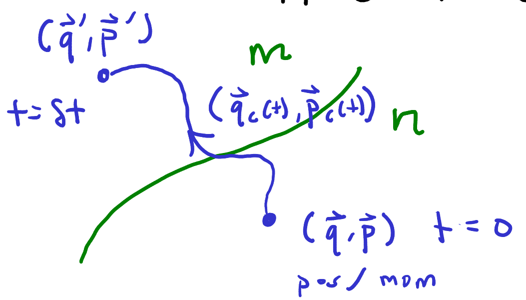
$\vec{p}(t_{i+1}) = W\,\vec{p}(t_i)$ DTDS
master equ.

also know: $t \to \infty$ $\quad p_\mu(t) \to \frac{1}{\Theta_{tot}}$

W must have this stationary state!
$\downarrow$
What does this say about W?

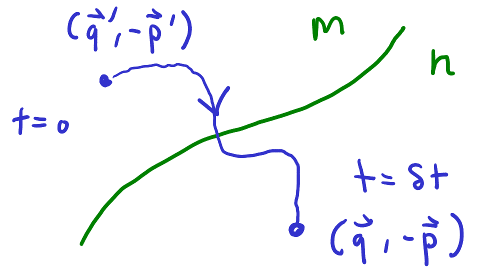
$$P_n(t) \to \frac{\Theta_n}{\Theta_{tot}} \equiv P_n^s \quad \text{stationary state}$$

use a property of classical mechanics: time reversal symmetry $\quad m \neq n$



traj. satisfies Hamilton's equ's:

$$\frac{d\vec{q}_c}{dt} = \frac{\partial H}{\partial \vec{p}_c} \qquad \frac{d\vec{p}_c}{dt} = -\frac{\partial H}{\partial \vec{q}_c}$$



claim: another "reversed" sol'n must also exist:

$$\vec{q}_r(\tilde{t}) \equiv \vec{q}_c(t-\tilde{t})$$
$$\vec{p}_r(\tilde{t}) = -\vec{p}_c(t-\tilde{t})$$

Proof:
$$\frac{d\vec{q}_c}{dt} = \frac{\partial \mathcal{H}}{\partial \vec{p}_c} \Rightarrow -\frac{d\vec{q}_r}{dt} = -\frac{\partial \mathcal{H}}{\partial \vec{p}_r} \quad \checkmark$$

$$\frac{d\vec{q}_c(t-\tilde{t})}{dt} = -\frac{d\vec{q}_c(t-\tilde{t})}{d\tilde{t}} = -\frac{d\vec{q}_r}{dt}$$

$$\frac{d\vec{p}_c}{dt} = -\frac{\partial \mathcal{H}}{\partial \vec{q}_c} \Rightarrow \frac{d\vec{p}_r}{d\tilde{t}} = -\frac{\partial \mathcal{H}}{\partial \vec{q}_r} \quad \checkmark$$

as $t \to \infty$ we approach stationary state where all microstates are equally likely

$\Rightarrow$ equally likely to start at $(\vec{q}, \vec{p})$ as $(\vec{q}\,', -\vec{p}\,')$

$\Rightarrow$ equally likely to be observe original + reversed trajectories

$\Rightarrow$ prob. of observing $m \to n$ trans. = prob. of observing $n \to m$ trans.

local detailed balance (LDB)

$$\boxed{\underbrace{W_{nm}\, p_m^s}_{\text{prob. to observe } m \to n \text{ trans.}} = W_{mn}\, p_n^s}$$

W_{nm}: prob. of $m \to n$ trans. given start in m

p_m^s: prob. to be in m in stat. state

W_{mn}, p_n^s: " "

tells us something about the elements of W matrix