$$\text{LDB}: \quad \frac{W_{nm}}{W_{mn}} = e^{-\beta(E_n - E_m)} = \frac{\Theta_n}{\Theta_m} = \frac{\Theta(E_{tot} - E_n)}{\Theta(E_{tot} - E_m)}$$

$$\beta = \left.\frac{\partial \ln \Theta}{\partial E}\right|_{E_{tot}} = \frac{1}{k_B T}$$

$f(x - \Delta x) \approx f(x) - \left.\frac{\partial f}{\partial x}\right|_x \Delta x + \dots$

$$\ln \Theta_n = \underbrace{\ln \Theta(E_{tot} - E_n)}$$
$$\approx \ln \Theta(E_{tot}) - \beta E_n$$

$$p_n^s = \frac{\Theta_n}{\Theta_{tot}} = \frac{\Theta_n}{\sum_m \Theta_m}$$

$$= \frac{e^{\ln \Theta_n}}{\sum_m \Theta_m} = \underbrace{\frac{\Theta(E_{tot})}{\sum_m \Theta_m}}_{\equiv 1/Z \text{ const. indep. of } n} e^{-\beta E_n} \Rightarrow \boxed{p_n^s = \frac{e^{-\beta E_n}}{Z}}$$

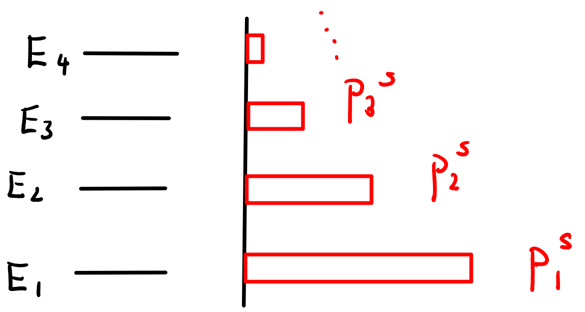
Boltzmann equil. distrib.

Z = partition function = normaliz. const.

$$\sum_n p_n^s = 1 \Rightarrow Z = \sum_m e^{-\beta E_m}$$

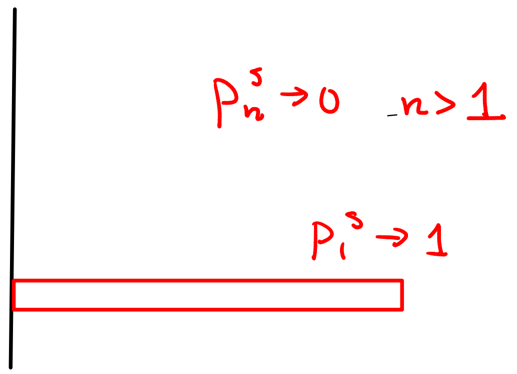
$\beta > 0$

$E_1 < E_2 < \cdots$



$T \to 0^+$

$\beta \to \infty$

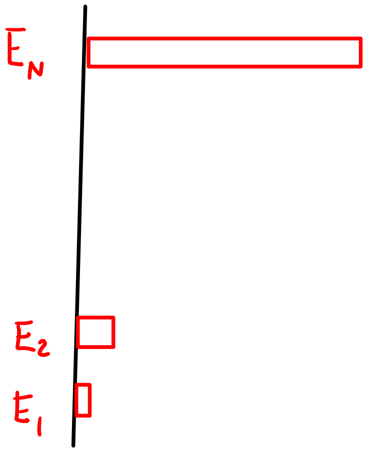


$$p_1^s = \frac{e^{-\beta E_1}}{e^{-\beta E_1} + e^{-\beta E_2} + \dots} \xrightarrow[\beta \to \infty]{} 1 \quad \text{as} \quad \beta \to \infty$$

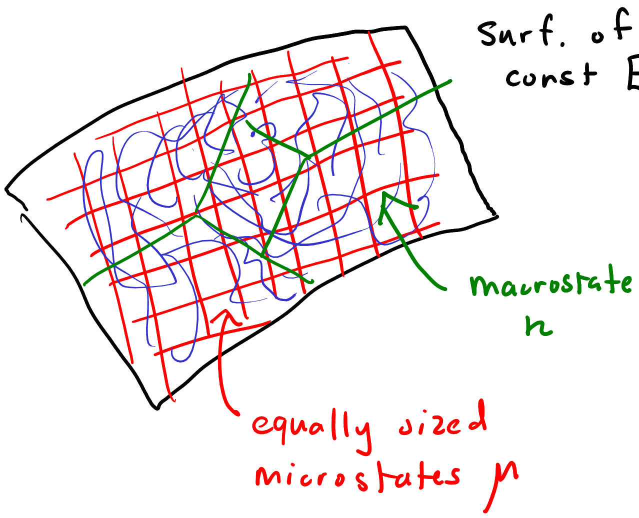
$$p_2^s = \frac{e^{-\beta E_2}}{e^{-\beta E_1} + e^{-\beta E_2} + \dots} \xrightarrow[\beta \to \infty]{} e^{-\beta(E_2 - E_1)} \to 0$$

$\beta \to 0^+ \quad T \to \infty$: $\quad p_n^s = \frac{1}{N}$ all macro-states equally likely

$\beta < 0$:



only way this works for $\beta < 0$ is for system to have a max. possible energy (quantum phen.)



two ways of describing $t \to \infty$ limit:

$$P_\mu(t) \xrightarrow[t\to\infty]{} \frac{1}{\Theta_{tot}}$$ microcanonical ensemble

$$P_n(t) \xrightarrow[t\to\infty]{} \frac{e^{-\beta E_n}}{Z}$$ canonical ensemble

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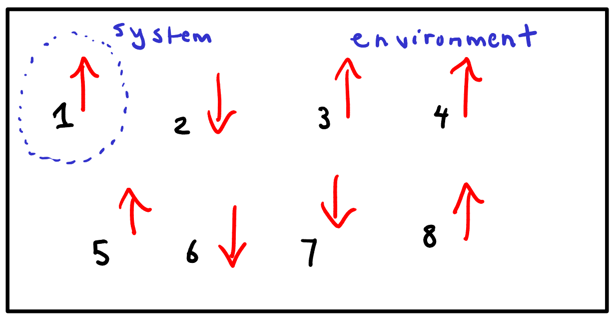
Example: "gas" of spins



in the total: k $\uparrow$ spins

$M+1-k$ $\downarrow$ spins

$M+1$ total spins

$k = 5$

$M+1 = 8$

energies: 0 if $\downarrow$

ϵ if $\uparrow$ $\quad \epsilon > 0$

$E_{tot} = k\epsilon$

system : spin 1

environment : all the other spins

system states $(N=2)$: $E_1 = 0$ (spin 1 = $\downarrow$)

$E_2 = \epsilon$ (spin 1 = $\uparrow$)

environ. energies: $E_1^{env} = E_{tot} - E_1 = k\epsilon$

$$E_2^{env} = E_{tot} - E_2 = (k-1)\epsilon$$

dynamics: at every time step δt

one "collision" occurs:

choose at random one $\downarrow$ spin

+ one $\uparrow$ spin + then flip the pair

$\Rightarrow$ preserves $k = \#\ \uparrow$ spins +

hence $E_{tot} = k\epsilon$ is const.

ergodic?

	1	2	3	....	M+1
initial state ($k\ \uparrow$ spins)	$\uparrow$	$\downarrow$	$\downarrow$	....	$\uparrow$
final state ($k\ \uparrow$ spins)	$\downarrow$	$\downarrow$	$\downarrow$	....	$\uparrow$