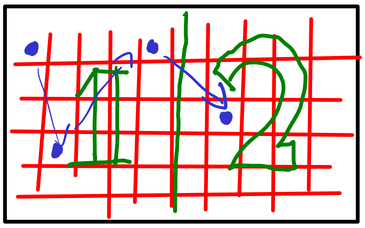
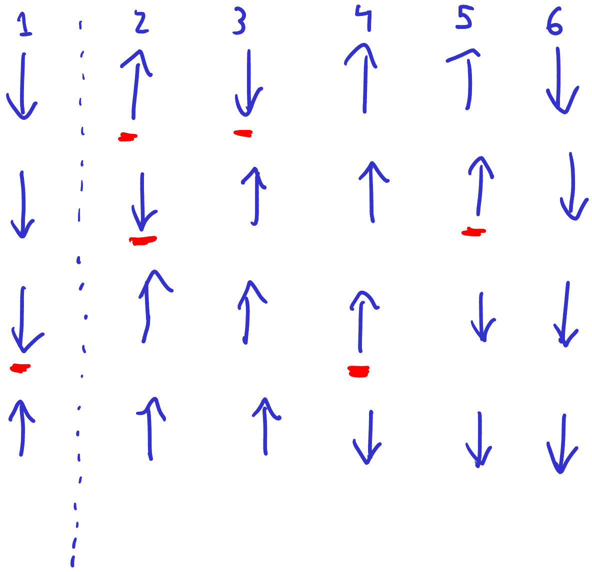


sys. state 1
spin 1 = ↓
" " 2
spin 1 = ↑



$k = 3$

every ↑ spin has energy ϵ
" ↓ spin " " 0

How big are the macrostates?

⇒ how many microstates (spin configs) exist for each sys. state?

$$\begin{array}{c|cccc} 1 & 2 & \cdots & & M+1 \\ \downarrow & \uparrow & \downarrow & \cdots & \downarrow \end{array}$$
$\underbrace{\qquad\qquad\qquad}_{\text{put } k \text{ up spins in } M \text{ slots}}$

sys. state 1

$$\Theta_1 = \binom{M}{k} = \frac{M!}{(M-k)!\,k!}$$

"size of country 1"

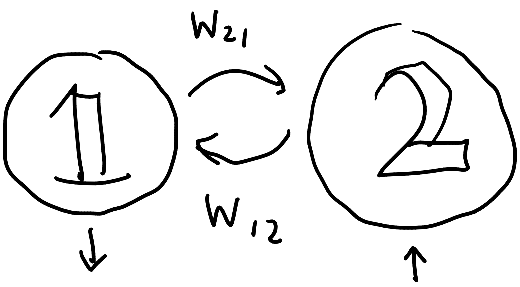
$$\begin{array}{c|cccc} 1 & 2 & \cdots & & M+1 \\ \uparrow & & & & \end{array}$$
$\underbrace{\qquad\qquad\qquad}_{\text{put } k-1 \text{ up spins in } M \text{ slots}}$

sys. state 2

$$\Theta_2 = \binom{M}{k-1} = \frac{M!}{(M-k+1)!\,(k-1)!}$$

"size of country 2"

transition matrix between macrostates



W_{21} = prob. that in time step δt we have spin 1 goes to ↑, given that it started ↓

$$W_{12} = \frac{1}{\#\uparrow \text{spins}} = \frac{1}{k} \qquad = \frac{1}{\#\downarrow \text{spins}} = \frac{1}{M+1-k}$$

check if LDB works: $\frac{W_{12}}{W_{21}} \stackrel{?}{=} \frac{p_1^s}{p_2^s} = \frac{\Theta_1}{\Theta_2}$

$$p_1^s = \frac{\Theta_1}{\Theta_{tot}} \qquad p_2^s = \frac{\Theta_2}{\Theta_{tot}}$$

$$= \frac{(M-k+1)!\,(k-1)!}{(M-k)!\;k!}$$

$$= \frac{M-k+1}{k} \quad \checkmark$$

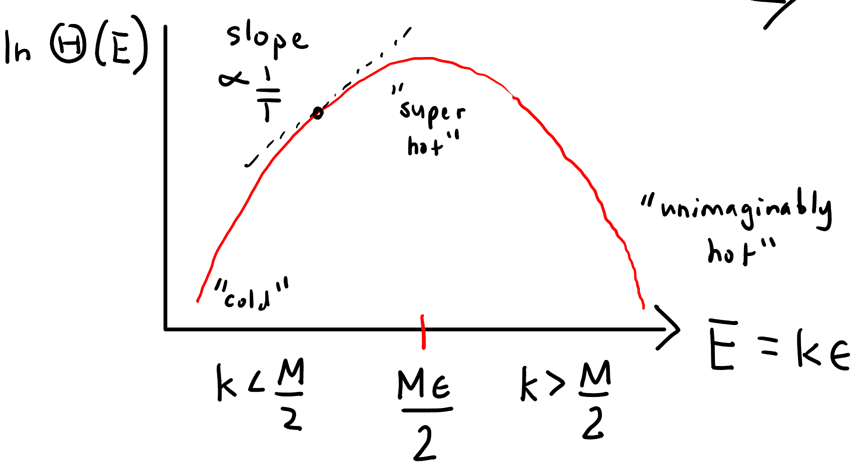
$$E_{tot} = k\epsilon$$

Derive temperature: $\Theta_n = \Theta(E_{tot} - E_n)$

$$E_{tot} - E_1 = E_1^{env} = E_{tot} \qquad \Theta_1 = \binom{M}{k} = \binom{M}{E_{tot}/\epsilon}$$

$$E_{tot} - E_2 = E_2^{env} = E_{tot} - \epsilon \qquad \Theta_2 = \binom{M}{k-1} = \binom{M}{\frac{E_{tot}-\epsilon}{\epsilon}}$$

$$\Rightarrow \Theta(E) = \binom{M}{E/\epsilon} \quad \text{universal function}$$



$$\beta = \frac{1}{k_B T} = \left.\frac{\partial \ln \Theta}{\partial E}\right|_{E_{tot}}$$

approx valid $1 \ll k \ll N$:

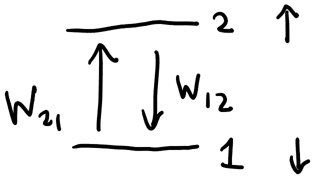
$$\ln \Theta(E) \approx \text{const.} - \frac{\left(M - \frac{2E}{\epsilon}\right)^2}{M}$$

$$\beta \approx \frac{2}{\epsilon}\left(1 - \frac{2E_{tot}}{M\epsilon}\right) \qquad E_{tot} = k\epsilon$$

$$k < \frac{M}{2} \Rightarrow \beta > 0,\ T > 0 \qquad k > \frac{M}{2} \Rightarrow \beta < 0,\ T < 0$$

physical interpretation:

$$\text{LDB} \qquad \frac{W_{12}}{W_{21}} = e^{-\beta(\overset{0}{E_1} - \overset{\epsilon}{E_2})} = e^{\beta\epsilon}$$



$\beta > 0$: $W_{12} > W_{21}$ downhill > uphill

sys. more likely to go from ↑ to ↓ than the reverse

(↑ spins in minority: spin $\underline{1}$ ↑ is more likely to get chosen)

$\beta < 0$: $W_{12} < W_{21}$ downhill < uphill

sys. more likely to go from ↓ to ↑ because ↓ spins in minority