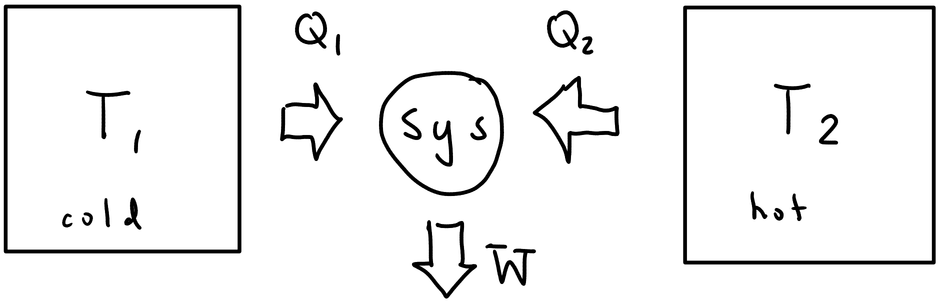


$T_2 > T_1 > 0$

$\dot{Q}_1 = \dfrac{Q_1}{\Delta t}$

$\dot{Q}_2 = \dfrac{Q_2}{\Delta t}$

$\sigma = \dfrac{I}{\Delta t} \geq 0$ $\qquad \dot{Q}_\alpha = \dfrac{Q_\alpha}{\Delta t}$, $\alpha = 1, 2$ $\qquad P = \dfrac{\bar{W}}{\Delta t}$ $\qquad \Delta t = \begin{cases} \delta t & \text{stat. state} \\ \tau \delta t & \text{periodic driving} \end{cases}$

Case 1: $\dot{Q}_2 > 0 \Rightarrow \dfrac{P}{\dot{Q}_2} = \eta = 1 - \dfrac{T_1}{T_2} - T_1 \sigma$

$\leq 1 - \dfrac{T_1}{T_2} \equiv \eta_{max}$ Carnot bound

heat engine:
draws heat from hot bath, dumps into cold bath ($\dot{Q}_1 < 0$) while doing some work ($\bar{W} > 0$)

max. efficiency $\eta \to \eta_{max}$ requires $\sigma = \dfrac{I}{\Delta t} \to 0$

two options:

- $I \to 0$ (equilibrium)

  set $T_1 = T_2$ + let sys. equil. at one temp.

  $\Rightarrow \eta = \eta_{max} = 1 - \dfrac{T_1}{T_2} = 0$

- $\Delta t \to \infty$ (infinitely long period of driving)

  $I \neq 0 \Rightarrow \bar{W} = Q_1 + Q_2 > 0$

  power $\dfrac{\bar{W}}{\Delta t} = P \to 0$ max. efficiency but zero power

more interesting question: what is efficiency at max. power
$\Rightarrow$ no universal answer

case 2: $\dot{Q}_1 > 0$ (draws heat from cold bath)

1st: $P = \dot{Q}_1 + \dot{Q}_2$

2nd: $\sigma = -\frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2} \geq 0$ $\longrightarrow$ $\frac{\dot{Q}_2}{T_2} = -\frac{\dot{Q}_1}{T_1} - \sigma$

$\Rightarrow \dot{Q}_2 < 0$ (dumps heat into hot bath)

example: refrigerator
heat pump

$$\dot{Q}_2 = -\frac{T_2}{T_1}\dot{Q}_1 - T_2\sigma \qquad \frac{T_2}{T_1} > 1$$

$$\Rightarrow |\dot{Q}_2| > |\dot{Q}_1|$$

$$\Rightarrow P = \dot{Q}_1 + \dot{Q}_2 = \dot{Q}_1 - |\dot{Q}_2| < 0$$

$\Rightarrow$ plug in your fridge

coeff. of performance $\eta_R = \frac{\dot{Q}_1}{-P} = \frac{\text{heat rate out cold bath}}{\text{input power}}$

$$\eta_R = \frac{\dot{Q}_1}{-\dot{Q}_1 - \dot{Q}_2} = \text{algebra} = \frac{T_1}{T_2 - T_1} - \underbrace{\frac{T_1 T_2 \sigma}{T_2 - T_1}}_{\leq 0}$$

$$\eta_R \leq \frac{\overset{\sim 300\,K}{T_1}}{\underset{\sim 30\,K}{T_2 - T_1}} = \eta_R^{max}$$

η_R^{max} can be quite large $\sim O(10)$

Big picture perspective: "many children of IFT"

IFT

- 2nd law consequences (max. S or min. F) [const. envir., one temp]
- Jarzynski equality [varying env., one temp]
- bounds on heat engine/pump performance [varying env., multiple temps]
- systems near equilibrium (small I) $\Downarrow$ linear response theory

Linear thermodynamics

fluctuation-dissipation theorem

Onsager reciprocity

[const. env., single temp., but out of equil.]

Single temp. T + stationary state (not necessarily equil.)

2nd: $\quad I = -\frac{1}{T}\cancel{\Delta F}^{\,0} - \frac{1}{T}\overline{W} \geq 0$

1st: $\quad Q = \cancel{\Delta E}^{\,0} + \overline{W}$

$$\Rightarrow \quad I = -\frac{1}{T}\overline{W} = -\frac{1}{T}Q \Rightarrow \underbrace{Q}_{\substack{\leq 0 \\ \text{heat into env.}}} = \underbrace{\overline{W}}_{\substack{\leq 0 \\ \text{work on sys.}}} = -TI \leq 0$$

$I = 0 \Rightarrow$ ESS $\quad (Q = \overline{W} = 0)$

$I > 0 \Rightarrow$ NESS $\quad (Q = \overline{W} < 0)$

"near" equil: I small $\quad (Q = \overline{W}$ also small$)$

focus on one time step δt: $\quad \mu_0 = (n_0, n_1)$

stationary state: $\vec{p}^{\,s}$ $\quad\quad p^s_{n_0} \quad p^s_{n_1}$

$$I = \langle I(\mu_0) \rangle = -\frac{1}{T}\langle W(\mu_0)\rangle = -\frac{1}{T}\overline{W}$$

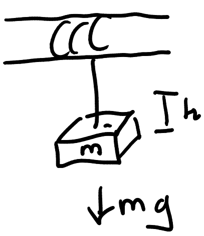
assume form for work: $\quad W(\mu_0) = \omega_{n_1 n_0}$

$$= -\underset{\substack{\uparrow \\ \text{"force"}}}{f}\ \underset{\substack{\uparrow \\ \text{"distance"}}}{\delta x_{n_1 n_0}}$$

"force" (some physical quantity we control)

"distance": change in a phys. variable

ratchet:  $\quad ① \rightleftarrows ② \rightleftarrows ③ \rightleftarrows \cdots$



$f = mg$

$$\delta x_{n_1 n_0} = \begin{cases} +h & \text{if } n_1 = n_0 + 1 \\ -h & \text{if } n_1 = n_0 - 1 \\ 0 & \text{otherwise} \end{cases}$$