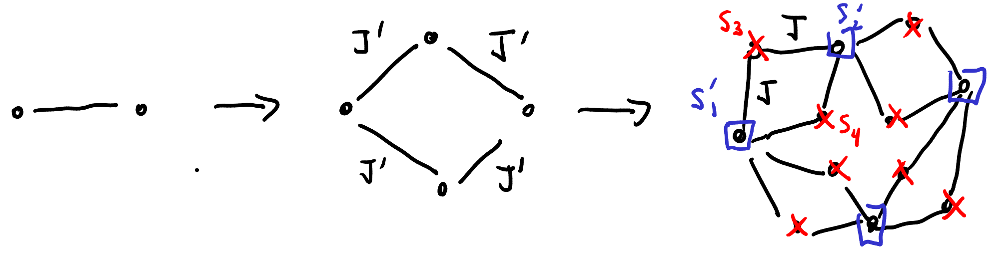
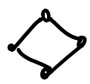


$b = 2$ = length rescaling

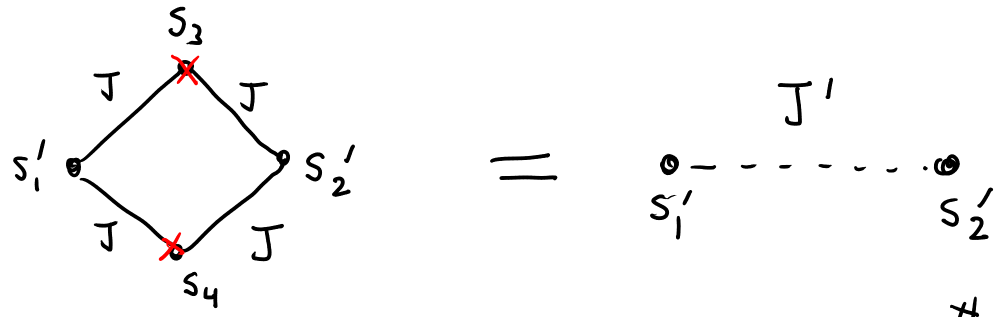
$d = 2$ dimension

$$Z = \sum_{\vec{s}} e^{-\beta \mathcal{H}_{\vec{s}}} = \sum_{\vec{s}\,'} \sum_{\vec{s}_{doc}} e^{-\beta \mathcal{H}_{\vec{s}}} \stackrel{?}{=} \sum_{\vec{s}\,'} e^{-\beta \mathcal{H}_{\vec{s}\,'}}$$

$$= \sum_{\vec{s}\,'} \left[\sum_{s_3, s_4} e^{J s_1' s_3 + J s_1' s_4 + J s_2' s_3 + J s_2' s_4} \right]$$

$\cdot$ [one term per ]

$\cdot$ [ ]



$$-\beta \mathcal{H}_{\vec{s}} = J \sum_{\langle ij \rangle} s_i s_j + G B$$

← # bonds

↑ addit. const. per bond

note: $G = 0$ in orig. system but $G \neq 0$ does not change physics

$$\Big\downarrow$$

$$-\beta \mathcal{H}_{\vec{s}\,'} = J' \sum_{\langle i'j' \rangle} s_i' s_j' + G' B'$$

$$B' = b^{-d} B = \frac{1}{4} B$$

for $Z = Z'$ to be true:

$$\sum_{\substack{s_3 = \pm 1 \\ s_4 = \pm 1}} e^{J s_1' s_3 + J s_1' s_4 + J s_2' s_3 + J s_2' s_4 + 4G} = e^{J' s_1' s_2' + G'}$$

must be true for every possible s_1', s_2'

$s_1'=1,\ s_2'=1$: $\quad e^{4J+4G} + 2e^{4G} + e^{-4J+4G} = e^{J'+G'}$

$s_1'=1,\ s_2'=-1$: $\quad 4e^{4G} = e^{-J'+G'}$

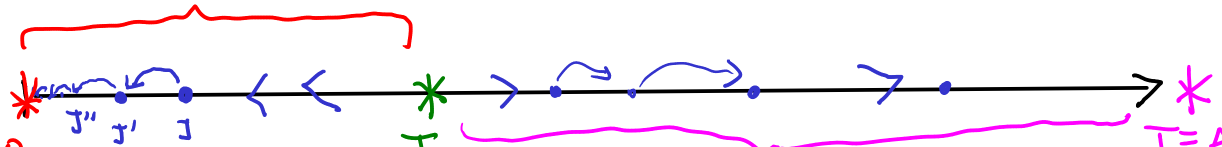
...
(same equ's) $\quad \Rightarrow$ 2 indep. equ's for 2 unknowns: J', G'

$$\Rightarrow \quad J' = \log\left[\frac{1}{2}\left(e^{2J}+e^{-2J}\right)\right]$$

$$G' = 4G + \log\left[2\left(e^{2J}+e^{-2J}\right)\right]$$

$J \to J' \to J'' \to \cdots$ RG flow for J parameter

basin of attraction for $J=0$ FP



$J=0$ fixed point (stable) [FP]

J_c fixed pt (unstable)

basin of attr. for $J=\infty$ FP

$J=\infty$ fixed pt (stable)

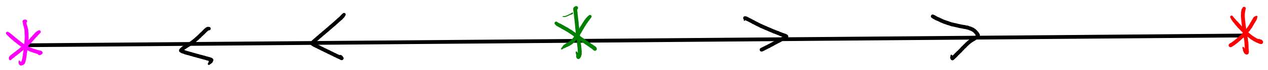
solution of $J_c = \log\left[\frac{1}{2}\left(e^{2J_c}+e^{-2J_c}\right)\right]$

$J_c \approx 0.609$

spins equally likely to be ↑ or ↓ (disordered)

all spins ↑ or all spins ↓

$J = \dfrac{j}{k_B T} \qquad T \propto J^{-1}$



$J^{-1}=0$

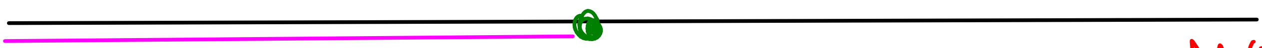
$J_c^{-1} \propto T_c$

$J^{-1}=\infty$

↕ one-to-one mapping of basins of attraction & phases

compare to:

T_c



ordered phase $M(J,0_\pm)\neq 0$ $\qquad$ disordered phase $M(J,0_\pm)=0$

boundaries b/t phases get their own FP

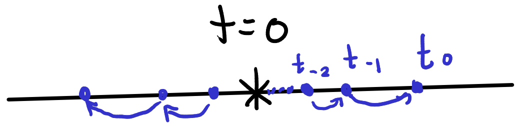
Zoom in around $T_c \propto J_c^{-1}$:

$$J = \frac{j}{k_B T}$$

$$t = \frac{T - T_c}{T_c} \quad \text{small}$$

$J = J_c$
$T = T_c$
$t = 0$

$$= \frac{J_c - J}{J} \approx \frac{J_c - J}{J_c}$$



$$J'(J) \approx J_c + \underbrace{\left.\frac{\partial J'}{\partial J}\right|_{J_c}}_{\equiv b^y} (J - J_c) + \dots$$

$J'(J_c) = J_c$

$$\Downarrow$$

$$J' - J_c = b^y (J - J_c)$$

$$y = \log_b \left.\frac{\partial J'}{\partial J}\right|_{J_c} = .747$$

$\Rightarrow \quad t' = b^y t \quad$ near crit. FP

$$t_0 = b^y t_{-1} = b^{2y} t_{-2} = \dots = b^{ny} t_{-n}$$

observables in system: deriv. of $\frac{1}{B} \ln Z$ or $\frac{1}{N} \ln Z$

example: spin-spin corr : $U = \frac{1}{B} \left\langle - \sum_{\langle ij \rangle} s_i s_j \right\rangle$

$$= - \frac{\partial}{\partial J} \left[\frac{1}{B} \ln Z \right]$$

$f(t) \equiv \frac{1}{B} \ln Z(t)$ $\quad$ near $T = T_c$ $\quad \propto \frac{\partial}{\partial t} f(t)$

free energy per bond

specific heat $\quad C = \left.\frac{dU}{dT}\right|_{H=0} \propto \frac{\partial^2}{\partial t^2} f(t)$

consider $Z(t)$ under RG:

$$\ln Z(t_0) = \ln Z(t_{-1}) = \cdots = \ln Z(t_{-n})$$

$$\frac{1}{B_0} \ln Z(t_0) = \frac{1}{B_0} \ln Z(t_{-n})$$

$B_n = b^{nd} B_0$
$= 4^n B_0$

$\uparrow$ # bonds in sys. at $t = t_0$

$$= b^{nd} \frac{1}{B_n} \ln Z(t_{-n})$$

in limit when ∞ sized sys.

$$f(t_0) = b^{nd} f(\overbrace{t_{-n}}^{t})$$

for any n

$\frac{1}{B_n} \ln Z(t_{-n})$
$= f(t_{-n})$

$\frac{1}{B_0} \ln Z(t_0)$
$= f(t_0)$

$$t_0 = b^{ny} t_{-n} = b^{ny} t$$

$$n = \frac{1}{y} \log_b \left(\frac{t_0}{t}\right)$$

$$\Rightarrow \quad f(t_0) = \left(\frac{t_0}{t}\right)^{\frac{d}{y}} f(t)$$

scaling relation for small t