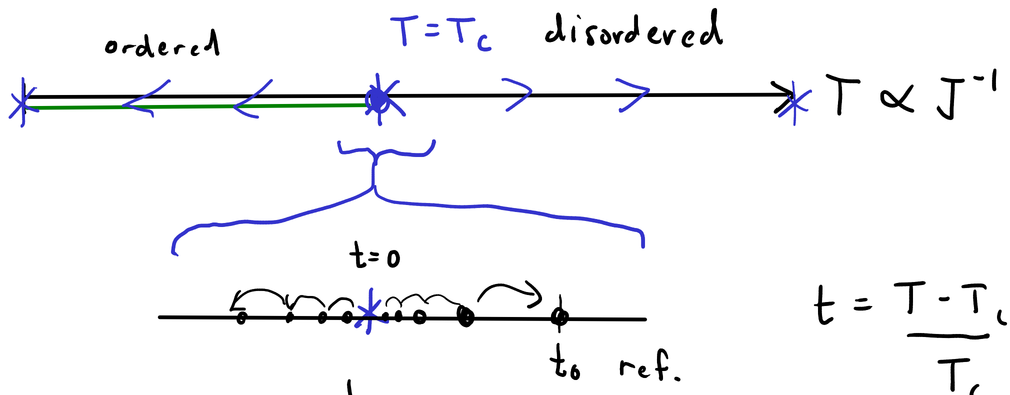


$$t = \frac{T - T_c}{T_c} \simeq \frac{J_c - J}{J_c}$$

$$f(t) = \frac{1}{\beta} \ln Z \qquad t_0 \ll 1$$

$$f(t_0) = \left(\frac{t_0}{t}\right)^{\frac{d}{y}} f(t) \qquad \left.\frac{\partial J'}{\partial J}\right|_{J_c} \equiv b^y \qquad b=2$$

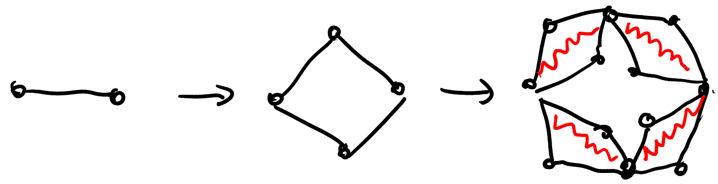
$$f(t) = \left(\frac{t}{t_0}\right)^{\frac{d}{y}} f(t_0)$$

Specific heat $\quad C \propto \frac{\partial^2}{\partial t^2} f \sim \left(\frac{t}{t_0}\right)^{\frac{d}{y}-2} \sim t^{-\alpha}$

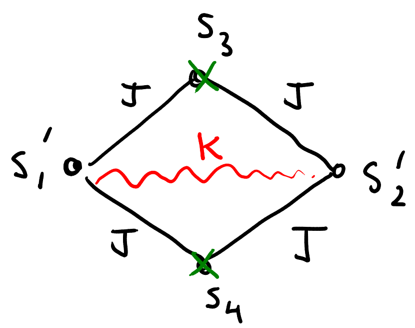
$$\Rightarrow \quad \alpha \equiv 2 - \frac{d}{y}$$

simple of universality: add a term to Ising model

n.n.n. interactions:



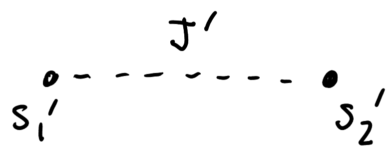
wwww = n.n.n.



wwww $= K s_1' s_2'$

$K > 0$

$\Rightarrow$

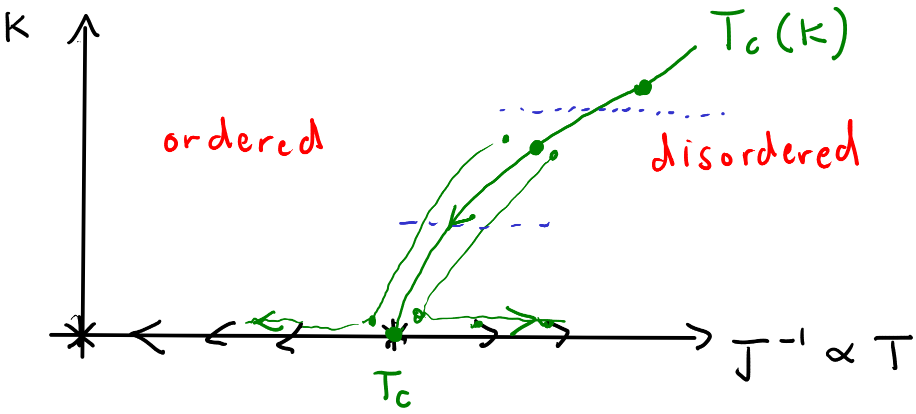


RG transf: $\quad K \neq 0 \quad K' = 0 \quad K'' = 0 \quad \ldots\ldots$

$$J' = K + \log\left(\tfrac{1}{2}\left(e^{2J} + e^{-2J}\right)\right) \qquad \text{1st step}$$

$$J'' = \log\left(\tfrac{1}{2}\left(e^{2J'} + e^{-2J'}\right)\right) \qquad \text{2nd step}$$

etc.



T_c inc. w/ K b/c n.n.n. favors ordered phase

phase trans. at finite K: scaling near $T_c(K)$ will be dominated by flow away from K=0 fixed point at $T_c(0)$ $\Rightarrow$ same exp. y as before

$\Rightarrow \quad \alpha = 2 - \frac{d}{y}$ same for any $K>0$

K is an example of an irrelevant term: adding it does not affect crit. exponents at phase boundary

---

Quantum stat. mechanics: $\quad \dot{\vec{p}}(t) = W(t)\,\vec{p}(t)$

analogue of classicable ensemble $p_n(t)$

prob. to be in classical state n at time t

$\Rightarrow$ quantum ensemble: many copies of a quantum system

operator: density oper.

$$\hat{\rho}(t) = \sum_n p_n |\psi_n\rangle\langle\psi_n|$$

p_n = fraction of ensemble in state $|\psi_n\rangle$ at time t

"prepared by exper." $\Rightarrow$ $\{|\psi_1\rangle, |\psi_2\rangle, \dots\}$ set does <u>not</u> have to be a complete basis, nor does it have to be orthogonal

we will assume $\langle\psi_n|\psi_n\rangle = 1$ for each n

example: qubit

$$\begin{array}{ll} \text{——} |1\rangle & E_1 \\ \text{——} |0\rangle & E_0 \end{array}$$

i) $\hat{\rho} = |\psi_1\rangle\langle\psi_1|$ $\qquad |\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

pure ensemble: $p_n = 1$ for some n & all others zero

ii) $\hat{\rho} = \frac{1}{4}|\psi_1\rangle\langle\psi_1| + \frac{3}{4}|\psi_2\rangle\langle\psi_2|$ $\qquad |\psi_2\rangle = |1\rangle$

mixed ensemble: $p_n < 1$ for all n

$\hat{\rho}$ can be written as a matrix in an orthormal basis, i.e. $\{|0\rangle, |1\rangle\}$

$\rho_{ij} = \langle i|\hat{\rho}|j\rangle \Rightarrow$ i) $\rho = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

ii) $\rho = \begin{pmatrix} \frac{1}{8} & \frac{1}{8} \\ \frac{1}{8} & \frac{7}{8} \end{pmatrix}$

$\langle 0|\hat{\rho}|0\rangle = \langle 0|\left\{ \frac{1}{4}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)\left(\frac{1}{\sqrt{2}}\langle 0| + \frac{1}{\sqrt{2}}\langle 1|\right) + \frac{3}{4}|1\rangle\langle 1|\right\}|0\rangle$

observable, i.e. $\hat{H}$ + we choose basis to be e-vecs of $\hat{H}$, then matrix elements are related to avg. of $\hat{H}$ in ensemble:

$$\langle H \rangle = \mathrm{tr}(\hat{\rho}\hat{H}) \qquad \hat{H}|i\rangle = E_i|i\rangle$$

$$= \sum_i \langle i|\hat{\rho}\hat{H}|i\rangle$$

diag. elem's of ρ matrix

$$\hat{\rho} = \sum_n p_n |\psi_n\rangle\langle\psi_n| \qquad = \sum_i E_i \underbrace{\langle i|\hat{\rho}|i\rangle}_{\rho_{ii}}$$

compare it to: $\langle E \rangle = \sum_n E_n p_n$

$$\rho_{ii} = \sum_n p_n |\langle i|\psi_n\rangle|^2 \qquad = \sum_n \sum_i E_i \underbrace{p_n}_{\substack{\text{prob.} \\ \text{of } |\psi_n\rangle \\ \text{in our ensemble}}} \underbrace{|\langle i|\psi_n\rangle|^2}_{\substack{\text{prob. of finding} \\ E_i \text{ when measuring} \\ \hat{H} \text{ in state } |\psi_n\rangle}}$$

$|\psi_n\rangle \Rightarrow$ measurement of $\hat{H}$

$|\langle i|\psi_n\rangle|^2 \Rightarrow$ prob. of outcome E_i

$\Rightarrow$ two contributions to prob.

classical: state n $\longleftrightarrow$ energy E_n

quantum: $|\psi_n\rangle$ $\begin{cases} E_1 & \text{prob. } |\langle 1|\psi_n\rangle|^2 \\ E_2 & |\langle 2|\psi_n\rangle|^2 \\ \vdots & \vdots \\ E_N & \vdots \end{cases}$

$$\rho_{ii} = \sum_n \overset{\geq 0}{p_n} \underbrace{|\langle i|\psi_n\rangle|^2}_{\geq 0} \geq 0 \quad \text{in any basis}$$

= total prob. to observe E_i in ensemble

$$\sum_i \rho_{ii} = \mathrm{tr}(\hat{\rho}) = \sum_{n,i} p_n \langle\psi_n|i\rangle\langle i|\psi_n\rangle$$

$$= \sum_n p_n = 1$$