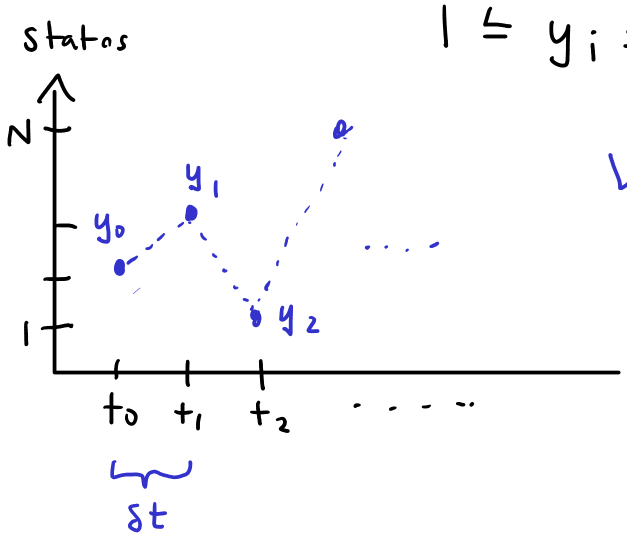
system variable $y(t)$ = labels macrostate of system at time $t$ (could be vector)

discretize time $t_i = i\,\delta t \qquad i = 0, 1, 2, \ldots$

$y(t) \Rightarrow y(t_i)$ = integer labeling macrostate (discrete "addresses")

$\equiv y_i$

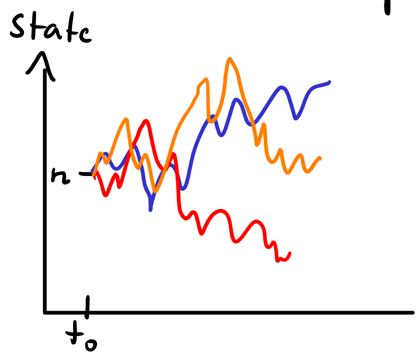
$1 \le y_i \le N$ = # macrostates




$\nu = (y_0, y_1, \ldots\ldots, y_i)$

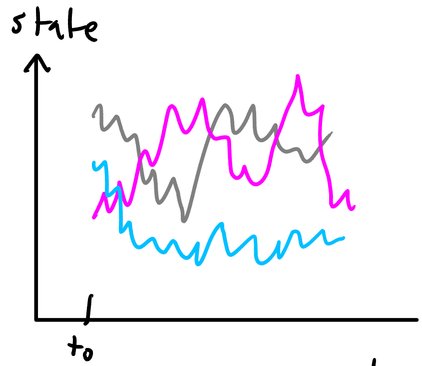
= trajectory associated w/ single exper. run

ensemble = collection of traj. from many exper. runs



pure ensemble

initial prob. dist. of states $\mathcal{P}(y_0) = \delta_{y_0,n} = \begin{cases} 1 & y_0 = n \\ 0 & y_0 \ne n \end{cases}$



mixed ensemble: $\mathcal{P}(y_0)$ is <u>not</u> a delta

$\mathcal{P}(\nu) = \mathcal{P}(y_0, y_1, \ldots, y_i) = \dfrac{\#\text{ traj. w/ seq. } (y_0, y_1, \ldots, y_i) \text{ in ensemble}}{\phantom{x}}$

prob. of observing $\nu$ in ensemble

total # of traj's $= N_{traj}$ (assume large)

$$\sum_{\nu} \mathcal{P}(\nu) = \sum_{y_0=1}^{N} \sum_{y_1=1}^{N} \cdots \sum_{y_i=1}^{N} \mathcal{P}(y_0, y_1, \ldots, y_i) = 1$$

sum over all possible traj's

physical quantity $Q(\nu)$ assoc. w/ traj. $\nu$

ensemble avg. $\langle Q \rangle \equiv \sum_{\nu} Q(\nu) \mathcal{P}(\nu)$

example: $Q(\nu) = E_{y_i} - E_{y_0}$ (total energy change)

$\langle Q(\nu) \rangle$ = avg. total energy change

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Review of basic probability concepts:

- underlying ensemble
- let A + B be "events" drawn from ensemble

example: $A = y_3$ state at time $t_3$

<u>or</u> $A = (y_0, y_1, y_2)$

$$\mathcal{P}(A) = \frac{\#\ \text{traj where A occurs}}{N_{traj}}$$

joint prob $\mathcal{P}(A, B) = \dfrac{\#\ \text{traj. where A + B occur together}}{N_{traj}}$

marginal prob. $\mathcal{P}(A) = \sum_B \mathcal{P}(A,B)$

$$\mathcal{P}(B) = \sum_A \mathcal{P}(A,B)$$

all prob's sum to 1: $\sum_A \mathcal{P}(A) = 1$ $\quad \sum_{A,B} \mathcal{P}(A,B) = 1$

$$\sum_B \mathcal{P}(B) = 1$$

conditional prob's: prob. of A given that B occurs

$$\mathcal{P}(A \mid B) = \frac{\#\text{ traj. where } A \text{ \& } B \text{ occur together}}{\#\text{ traj. where } B \text{ occurs}}$$

$$= \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)}$$

note: $\sum_A \mathcal{P}(A \mid B) = \sum_A \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)} = \frac{1}{\mathcal{P}(B)} \mathcal{P}(B) = 1$

$\sum_B \mathcal{P}(A \mid B) = \sum_B \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)} \neq 1$ in general

A, B can be in past / future relative to each other

if A & B are independent: $\mathcal{P}(A,B) = \mathcal{P}(A)\mathcal{P}(B)$

$$\mathcal{P}(A \mid B) = \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)} = \mathcal{P}(A)$$

$$\mathcal{P}(B \mid A) = \mathcal{P}(B)$$

another property: Bayes theorem

$$\boxed{\mathcal{P}(A \mid B) = \frac{\mathcal{P}(B \mid A)\,\mathcal{P}(A)}{\mathcal{P}(B)}}$$

Proof: $\mathcal{P}(B \mid A) = \dfrac{\mathcal{P}(A, B)}{\mathcal{P}(A)}$

plug in RHS of Bayes: $\dfrac{\mathcal{P}(A, B)}{\mathcal{P}(B)} = \underset{\text{LHS}}{\mathcal{P}(A \mid B)}$

example: 1) statement: "I have 3 kids & at least one is a boy."

What is the prob. that I have 3 boys?

$\tilde{B}$ = at least 1 is a boy

BBB = all three are boys

$\mathcal{P}(BBB \mid \tilde{B}) = ?$

all possible trajectories:

$$\left.\begin{matrix} BBB \\ BBG \\ BGB \\ GBB \\ GGB \\ GBG \\ BGG \end{matrix}\right\} \tilde{B} \text{ is true in 7 traj.} \qquad \mathcal{P}(BBB \mid \tilde{B}) = \frac{1}{7}$$

GGG

Check Bayes: $\mathbb{P}(BBB \mid \tilde{B}) = \dfrac{\mathbb{P}(\tilde{B} \mid BBB)\,\mathbb{P}(BBB)}{\mathbb{P}(\tilde{B})}$

$$= \frac{1 \cdot \frac{1}{8}}{\frac{7}{8}} = \frac{1}{7}$$