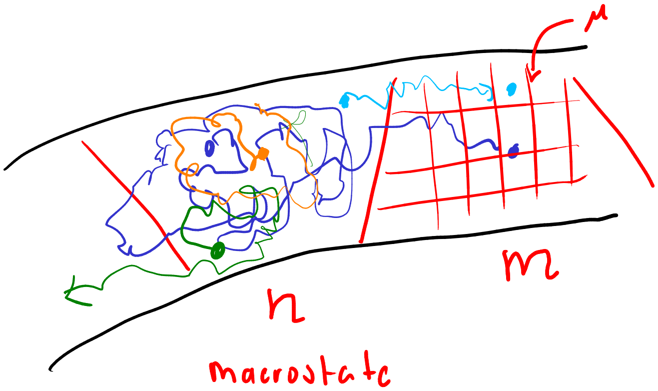


define transition matrix:

W_{mn} = prob. to end up in m after time interval δt given start in n

$$= \frac{\#\text{ traj. that start in } n + \text{end up in } m \text{ after } \delta t}{\#\text{ '' '' '' '' } + \text{end up anywhere after } \delta t}$$

$p_n(t_i)$ = prob. of macrostate n at time t_i

$$\vec{p}(t_{i+1}) = W\, \vec{p}(t_i)$$

DTDS master equ.

know: $t \to \infty$

$$p_\mu(t) \to \frac{1}{\Theta_{tot}}$$

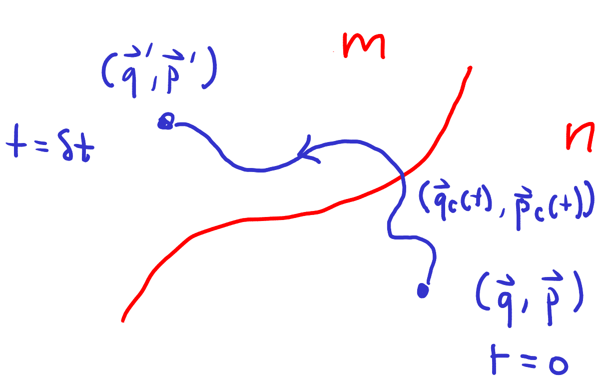
$$p_n(t) \to \frac{\Theta_n}{\Theta_{tot}}$$

Θ_n ← # boxes in country

Θ_{tot} ← total # boxe

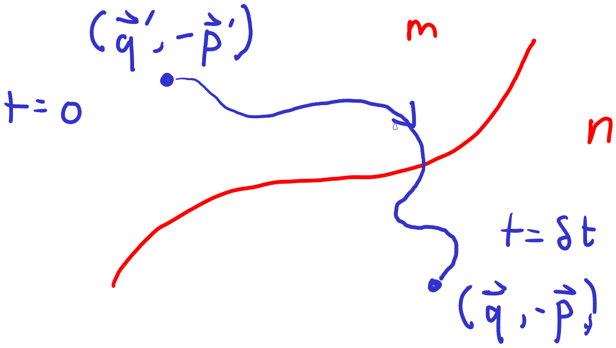
$\equiv p_n^s$ stationary state

Next: use time reversal symmetry



traj. satisfies Hamilton's equ:

$$\frac{d\vec{q}_c}{dt} = \frac{\partial H}{\partial \vec{p}_c} \qquad \frac{d\vec{p}_c}{dt} = -\frac{\partial H}{\partial \vec{q}_c}$$



"reversed" solution:

$$\vec{q}_r(\tilde{t}) \equiv \vec{q}_c(t-\tilde{t})$$
$$\vec{p}_r(\tilde{t}) \equiv -\vec{p}_c(t-\tilde{t})$$

claim: this also satisfies Hamilton's equ

proof:

$$\frac{d\vec{q}_c}{dt} = \frac{\partial \mathcal{H}}{\partial \vec{p}_c} \Rightarrow -\frac{d\vec{q}_r}{d\tilde{t}} = -\frac{\partial \mathcal{H}}{\partial \vec{p}_r} \quad \checkmark$$

$$\frac{d\vec{q}_c(t-\tilde{t})}{dt} = -\frac{d\vec{q}_c}{d\tilde{t}}(t-\tilde{t}) = -\frac{d\vec{q}_r}{d\tilde{t}}$$

$$\frac{d\vec{p}_c}{dt} = -\frac{\partial \mathcal{H}}{\partial \vec{q}_c} \Rightarrow \frac{d\vec{p}_r}{d\tilde{t}} = -\frac{\partial \mathcal{H}}{\partial \vec{q}_r} \quad \checkmark$$

argument: We approach stat. state as $t \to \infty$ where all microstates are equally likely

$t \to \infty$: $\Rightarrow$ equally likely to start at $(\vec{q}, \vec{p})$ as $(\vec{q}, -\vec{p})$

only true in this limit

$\Rightarrow$ equally likely to observe original & reversed trajectories

$\Rightarrow$ prob. of observing $m \to n$ trans. $=$ prob. of observing $n \to m$ trans.

local detailed balance (LDB)

$$\boxed{W_{nm}\, p^s_m = W_{mn}\, p^s_n}$$

W_{nm}: prob. of $m \to n$ given a start in m

p^s_m: prob. start in m

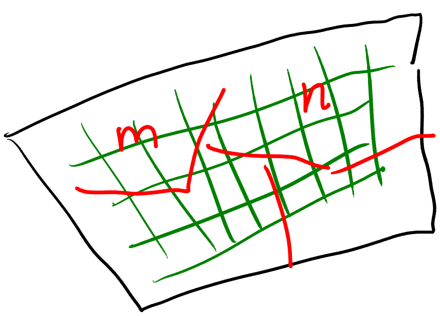
$W_{mn}\, p^s_n$: prob. of observing $n \to m$ trans.

$W_{nm}\, p^s_m$: prob. of observing $m \to n$ trans.

$\Rightarrow$ tells us an important property which elements of W matrix must satisfy

Zoom out:

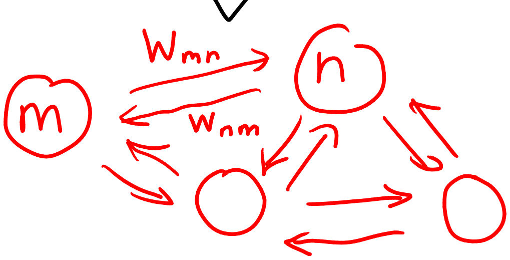
1) isolated total (system + environment) w/ total energy E_{tot}, which is ergodic + mixing



E_{tot}

2) divide up total into a "system" (focus) + "environment"

$\Updownarrow$



3) network of transitions W satisfying LDB:

$$W_{nm}\, p^s_m = W_{mn}\, p^s_n$$

$$p_n^s = \frac{\Theta_n}{\Theta_{tot}} \qquad p_m^s = \frac{\Theta_m}{\Theta_{tot}}$$

immediate consequence of LDB:

if $W_{nm} \neq 0 \Rightarrow W_{mn} \neq 0$

if arrow exists in one direction, has to exist in the other

$\Rightarrow$ microscopic reversibility (MR)

$\Rightarrow$ get a unique stationary state

Properties of this stationary state:

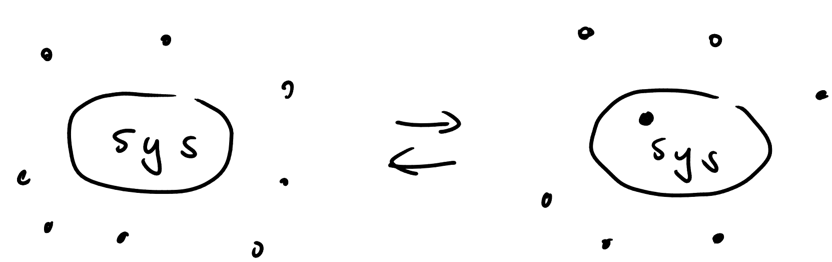
Θ_n = counts # of microstates available to environment when sys. is in state n

$\Rightarrow$ should depend on conserved quantities describing environment

examples: energy $\underset{\text{sys}}{E_{tot} = E_n} + \underset{\text{env.}}{E_n^{env}}$

$\Rightarrow E_n^{env} = E_{tot} - E_n$

exchange of particles



$$N^{part}_{tot} = N^{part}_{n} + N^{part,env}_{n}$$

\# part. in tot — sys — env.

$$\Rightarrow \quad \Theta_n = \Theta\left(E^{env}_n,\ N^{part,env}_n,\ \dots\right)$$

universal func.

other conserved quantities