$$p_n^s = \frac{e^{-\beta E_n}}{Z}$$

$$\beta = \frac{1}{k_B T} \to 0^+$$

$$T \to \infty$$

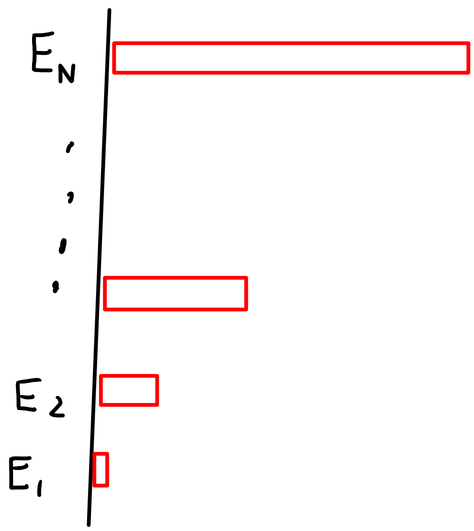
$$Z = \sum_m e^{-\beta E_m}$$

$$\to N$$

$$p_n^s \to \frac{1}{N}$$

all macrostates are equally likely

$\beta < 0$:



only way this works is for a system to have a max. possible energy (i.e. in some quantum contexts)

Example: "gas" of spins

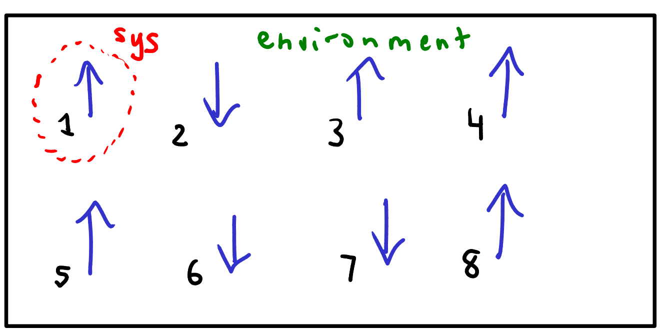
total:

k $\uparrow$ spins

$M+1-k$ $\downarrow$ spins

$M+1$ total spins

$E_{tot} = k\epsilon$



Spin 1 = sys

all other spins = env.

$k = 5$

$M+1 = 8$

energies: 0 if $\downarrow$

ϵ if $\uparrow$

$N = 2$ sys. states: $E_1 = 0$ (spin 1 = $\downarrow$)

$E_2 = \epsilon$ (spin 1 = $\uparrow$)

env. energies: $E_1^{env} = E_{tot} - E_1 = k\epsilon$

$E_2^{env} = E_{tot} - E_2 = (k-1)\epsilon$

dynamics: at every time step δt one "collision" occurs: choose at random one $\downarrow$ + one $\uparrow$ spin & flip the pair

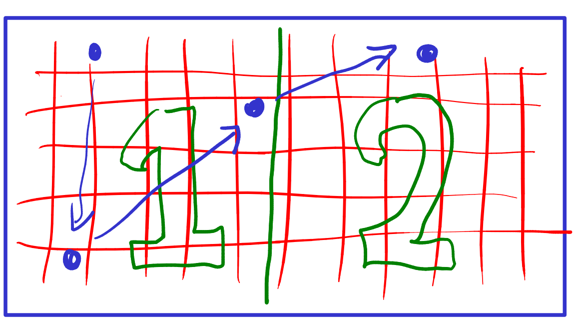
$\Longrightarrow$ preserves $k = \#\uparrow$ spins & hence $E_{tot} = k\epsilon$ is const.

ergodic & mixing

	1	2	3	....	M+1
initial state k ↑ spins	$\uparrow$	$\downarrow$	$\downarrow$	....	$\uparrow$

$\Downarrow$ can achieve thru some seq. of "collisions"

final state k ↑ spins	$\downarrow$	$\downarrow$	$\downarrow$	....	$\uparrow$



1	2	3	4	....	
$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$
$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$

$\uparrow \; \vdots \; \downarrow \quad \downarrow \quad \downarrow \quad \uparrow \quad \uparrow$

How big are the macrostates?

$\Rightarrow$ how many microstates (spin configs) exist for each sys. state?

$1 \; \vdots \; \underbrace{2 \cdots M+1}_{\text{put } k \text{ up spins in } M \text{ slots}}$
$\downarrow$

sys. state 1

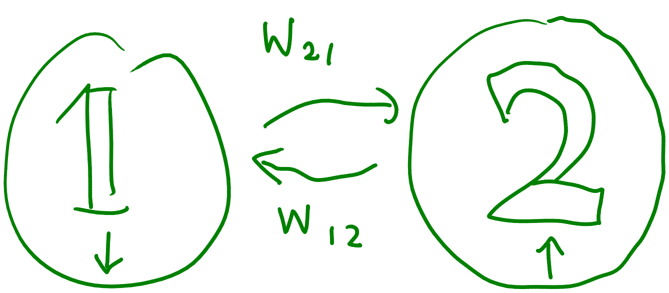
$$\Theta_1 = \binom{M}{k} = \frac{M!}{(M-k)!\,k!}$$

$1 \; \vdots \; \underbrace{2 \cdots M+1}_{\text{put } k-1 \uparrow \text{ spins in } M \text{ slots}}$
$\uparrow$

sys. state 2

$$\Theta_2 = \binom{M}{k-1} = \frac{M!}{(M-k+1)!\,(k-1)!}$$

transition matrix between macrostates



W_{21} = prob. that in time step δt spin 1 goes to $\uparrow$, given started in $\downarrow$

$$W_{21} = \frac{1}{\#\downarrow\text{ spins}} = \frac{1}{M+1-k}$$

$$W_{12} = \frac{1}{\#\uparrow \text{ spins}} = \frac{1}{k} \qquad \Theta_{tot} = \Theta_1 + \Theta_2$$

check if LDB works: $\quad p_1^s = \frac{\Theta_1}{\Theta_{tot}} \qquad p_2^s = \frac{\Theta_2}{\Theta_{tot}}$

$$W_{12}\, p_2^s = W_{21}\, p_1^s \qquad \frac{W_{12}}{W_{21}} = \frac{p_1^s}{p_2^s} = \frac{\Theta_1}{\Theta_2} = \frac{(M-k+1)!\,(k-1)!}{(M-k)!\;k!}$$

$$= \frac{M-k+1}{k} \quad \checkmark$$

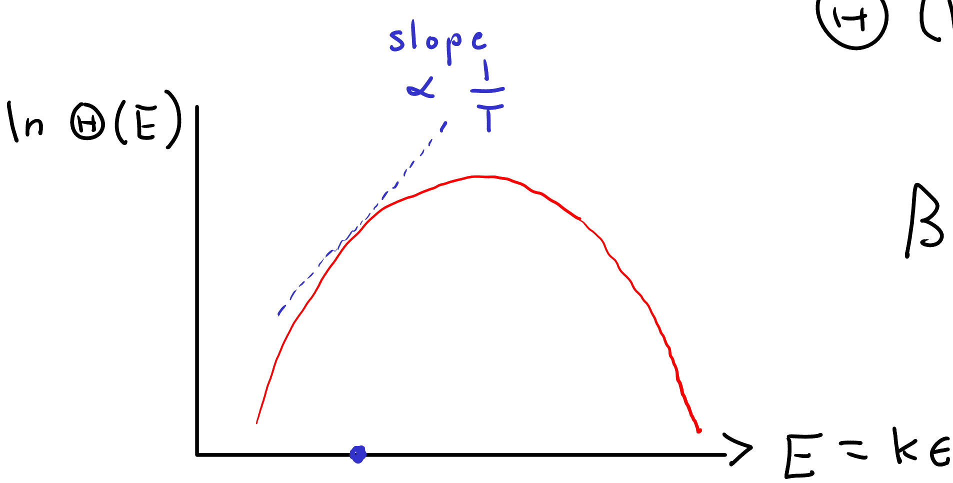
derive: temperature $\quad \Theta_n = \Theta(E_{tot} - E_n)$

$$E_{tot} - E_1 = E_1^{env} = E_{tot} \qquad \Theta_1 = \binom{M}{k} = \binom{M}{E_{tot}/\epsilon}$$

$$E_{tot} - E_2 = E_2^{env} = E_{tot} - \epsilon \qquad \Theta_2 = \binom{M}{k-1} = \binom{M}{\frac{E_{tot}-\epsilon}{\epsilon}}$$

$$E_{tot} = k\epsilon$$

$$\Theta(E) = \binom{M}{E/\epsilon} \quad \text{universal func.}$$



$$\beta = \frac{1}{k_B T} = \left.\frac{\partial \ln \Theta}{\partial E}\right|_{E_{tot}}$$

$$\ln \Theta(E) \approx \text{const.} - \frac{\left(M - \frac{2E}{\epsilon}\right)^2}{M}$$

$1 \ll k \ll N$

$$\beta \approx \frac{2}{\epsilon}\left(1 - \frac{2E_{tot}}{M\epsilon}\right) \qquad E_{tot} = k\epsilon$$