$$\nu = (n_0, n_1, n_2, \ldots\ldots, n_\tau)$$

$$I(\nu) = I(\mu_0) + I(\mu_1) + \cdots\cdots + I(\mu_{\tau-1}) = \mathrm{RHS}$$

$$\mu_0 = (n_0, n_1) \qquad \mu_1 = (n_1, n_2) \qquad \cdots\cdots$$

proof: $$I(\mu_i) = k_B \ln \frac{\mathcal{P}(\mu_i)}{\tilde{\mathcal{P}}(\tilde{\mu}_i)} = k_B \ln \frac{W_{n_{i+1} n_i} p_{n_i}(t_i)}{W_{n_i n_{i+1}} \underbrace{\tilde{p}_{n_{i+1}}(t_i)}_{p_{n_{i+1}}(t_{i+1})}}$$

$$\mu_i = (n_i, n_{i+1})$$

$$t_i \quad t_{i+1}$$

t_0 $\quad$ t_τ

$\vec{p}(t_0)$ $\quad$ $\vec{p}(t_\tau)$

$$\vec{p}(t_{i+1}) = W \vec{p}(t_i)$$

$$\tilde{p}(t_0) = \vec{p}(t_\tau) \qquad \tilde{p}(t_i) = \vec{p}(t_{i+1})$$

$$\mathrm{RHS} = k_B \ln \frac{W_{n_1 n_0} p_{n_0}(t_0)}{W_{n_0 n_1} \cancel{p_{n_1}(t_1)}} \cdot \frac{W_{n_2 n_1} \cancel{p_{n_1}(t_1)}}{W_{n_1 n_2} \cancel{p_{n_2}(t_2)}} \cancel{\cdots\cdots}$$

$$\cdots \frac{W_{n_\tau n_{\tau-1}} \cancel{p_{n_{\tau-1}}}(t_{\tau-1})}{W_{n_{\tau-1} n_\tau} p_{n_\tau}(t_\tau)}$$

$$= k_B \ln \frac{W_{n_\tau n_{\tau-1}} \cdots\cdots W_{n_1 n_0} p_{n_0}(t_0)}{W_{n_0 n_1} \cdots\cdots W_{n_{\tau-1} n_\tau} p_{n_\tau}(t_\tau)}$$

$$= k_B \ln \frac{\mathcal{P}(\nu)}{\tilde{\mathcal{P}}(\tilde{\nu})} = I(\nu)$$

net result: traj $\nu \Rightarrow I(\nu) = \sum_{i=0}^{I-1} I(\mu_i)$

IFT: $\langle e^{-I(\nu)/k_B} \rangle = 1 \Rightarrow \langle I(\nu) \rangle = I \geq 0$

Can $\langle I(\nu) \rangle = 0$?

focus on stat. state $\vec{p}^{\,s}$

$$p_n(t_i) \rightarrow p_n^s$$

$$I(\mu_i) = k_B \ln \frac{W_{n_{i+1} n_i}\, p^s_{n_i}}{W_{n_i n_{i+1}}\, p^s_{n_{i+1}}}$$

$$\langle I(\nu) \rangle = 0 = \sum_i \langle I(\mu_i) \rangle \qquad \langle I(\mu_i) \rangle \geq 0$$

$$I(\mu_i) = 0 \Leftrightarrow W_{n_{i+1}, n_i}\, p^s_{n_i} = W_{n_i n_{i+1}}\, p^s_{n_{i+1}}$$

LDB $\quad W_{ij}\, p_j^s = W_{ji}\, p_i^s$

If LDB is valid & we have reached stationary state: $I(\mu_i) = 0$ for every μ_i

$$I(\nu) = \sum_i I(\mu_i) = 0 \Leftarrow$$

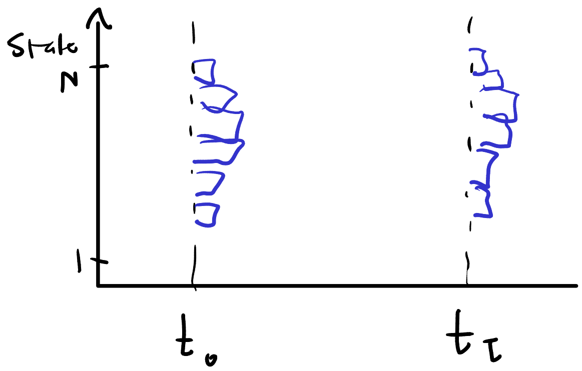
$\Rightarrow \langle I(\nu) \rangle = 0$ as well

$\Rightarrow$ known as: equilibrium stationary state (ESS)

If we are in a stationary state where $\langle I(\nu) \rangle > 0$

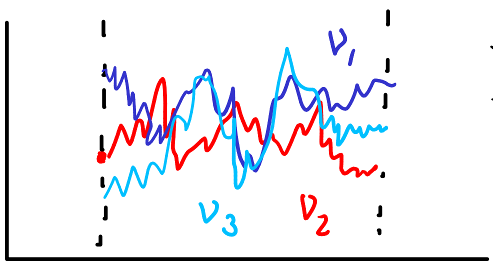
$\Rightarrow$: nonequilibrium stationary state (NESS)

Stationary state



$\Rightarrow$ how is this possible

$\Rightarrow$ can't exist if the form of LDB is valid



$I(\nu_1) = 0$

$I(\nu_2) = 0$

$I(\nu_3) > 0$

$\ldots$

$\mathcal{P}(\nu)$

= prob of $\nu = (n_0, \ldots, n_\tau)$ in the ensemble defined by $\vec{p}(t_0)$

$\tilde{\mathcal{P}}(\tilde{\nu})$ = prob. of $\tilde{\nu} = (n_\tau, \ldots, n_0)$ in the ensemble defined $\tilde{\vec{p}}(t_0) = \vec{p}(t_\tau)$

Upshot: for NESS to be possible it must be possible to modify the LDB condition in some systems

$\Rightarrow$ in order to get NESS we need to describe system coupling to external "work" $\Rightarrow$ modify LDB condition