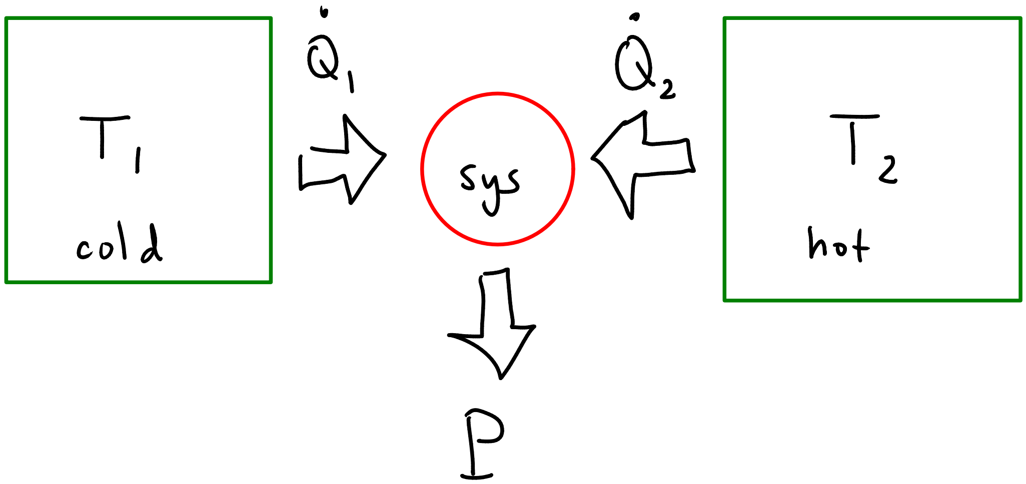


$T_2 \geq T_1$

$\sigma \equiv$ ent. prod. ≥ 0

case 1: $\dot{Q}_2 > 0 \Rightarrow \eta = \frac{P}{\dot{Q}_2} = 1 - \frac{T_1}{T_2} - T_1 \sigma$

$\Rightarrow \dot{Q}_1 < 0$

$\leq 1 - \frac{T_1}{T_2} \equiv \eta_{max}$ Carnot bound

case 2: $\dot{Q}_1 > 0$ (draw heat out of cold env.)

1st: $P = \dot{Q}_1 + \dot{Q}_2$

2nd: $\sigma = -\frac{\dot{Q}_1}{T_1} - \frac{\dot{Q}_2}{T_2} \geq 0 \rightarrow \frac{\dot{Q}_2}{T_2} = \underbrace{-\frac{\dot{Q}_1}{T_1} - \sigma}_{<0}$

(stat. state or cycle)

$\Rightarrow \dot{Q}_2 < 0$ dumping heat into hot envir.

example: refrigerators
heat pumps

$\dot{Q}_2 = -\frac{T_2}{T_1}\dot{Q}_1 - T_2 \sigma \qquad \frac{T_2}{T_1} > 1$

$\Rightarrow |\dot{Q}_2| > |\dot{Q}_1|$

$\Rightarrow P = \dot{Q}_1 + \dot{Q}_2 = \underbrace{\dot{Q}_1 - |\dot{Q}_2|}_{<0}$

$\Rightarrow$ needs net power

input into system

$\Rightarrow$ plug in your fridge

coeff. of performance $\eta_R = \frac{\dot{Q}_1}{-P} = \frac{\text{heat rate out of cold bath}}{\text{input power}}$

$$\eta_R = \frac{\dot{Q}_1}{-\dot{Q}_1 - \dot{Q}_2} = \text{algebra} = \frac{T_1}{T_2 - T_1} - \underbrace{\frac{T_1 T_2 \sigma}{T_2 - T_1}}_{\geq 0}$$

$$\eta_R \leq \frac{\overset{\sim 300K}{T_1}}{\underset{\sim 30K}{T_2 - T_1}} \equiv \eta_R^{max}$$

η_R^{max} can be quite large (on order ≈ 10)

Big picture perspective: "many children of IFT"

IFT

- 2nd law consequences (max. S or min. F) [const. env, one temp]
- Jarzynski equality [varying env., one temp]
- bounds on heat engines/pumps [multiple temps]
- systems near equilibrium (small I) $\Downarrow$ linear response theory

Linear thermodynamics

$\downarrow$ Onsager reciprocity

$\downarrow$ fluctuation-dissipation theorem

Single temp. T & stat. state (not necessarily equil.)

$$\text{2nd:} \quad I = -\frac{1}{T}\cancelto{0}{\Delta F} - \frac{1}{T}\overline{W} \geq 0 \qquad \cancelto{0}{\Delta F} = \cancelto{0}{\Delta E} - T\cancelto{0}{\Delta S}$$

$$\text{1st:} \quad Q = \cancelto{0}{\Delta E} + \overline{W}$$

$$I = -\frac{1}{T}\overline{W} = -\frac{1}{T}Q \Rightarrow \underbrace{Q}_{\substack{\leq 0 \\ \text{heat} \\ \text{into env.}}} = \underbrace{\overline{W}}_{\substack{\leq 0 \\ \text{work} \\ \text{on sys.}}} = -TI \leq 0$$

$I = 0 \Rightarrow$ ESS $(Q = \overline{W} = 0)$

$I > 0 \Rightarrow$ NESS $(Q = \overline{W} < 0)$

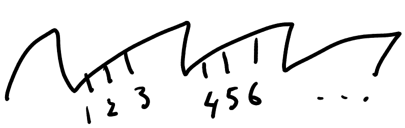
"near" equil: I small ($Q = \overline{W}$ is small)

focus on one time step δt: $\mu_0 = (n_0, n_1)$, with $n_0 \to P^s_{n_0}$, $n_1 \to P^s_{n_1}$

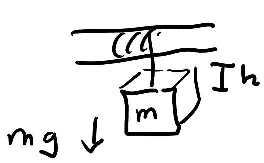
$$I = \langle I(\mu_0)\rangle = -\frac{1}{T}\overline{W} = -\frac{1}{T}\langle W(\mu_0)\rangle$$

assume form for work: $W(\mu_0) = \omega_{n_1 n_0}$

$$= -\underset{\substack{\uparrow \\ \text{"force"} \\ \text{(some phys. quantity we control)}}}{f}\ \underset{\substack{\uparrow \\ \text{"distance":} \\ \text{change in phys. obser.}}}{\delta x_{n_1 n_0}}$$



$\rightleftarrows (1) \rightleftarrows (2) \rightleftarrows (3) \rightleftarrows$



$$f = mg$$

$$\delta x_{n_1 n_0} = \begin{cases} +h & \text{if } n_1 = n_0 + 1 \\ -h & \text{if } n_1 = n_0 - 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f=0 \Rightarrow \overline{W}(\mu_0)=0 \Rightarrow \overline{W}=0 \Rightarrow \overline{I}=0$$

ESS

assume: $\delta x_{nm} = -\delta x_{mn} \qquad \delta x_{nn}=0$

when $f \neq 0$: $\overline{W} = -f\langle \delta x\rangle = -f \sum_{n_1,n_0} P(\mu_0)\,\delta x_{n_1 n_0}$

$$= -f \sum_{n_1,n_0} W_{n_1 n_0} p^s_{n_0}\, \delta x_{n_1 n_0}$$

LDB: $\dfrac{W_{n_1 n_0}}{W_{n_0 n_1}} = e^{-\beta(E_{n_1}-E_{n_0}-f\,\delta x_{n_1 n_0})}$

$W\vec{p}^{\,s} = \vec{p}^{\,s}$

$\Rightarrow$ both W + $\vec{p}^{\,s}$ (it's right e-vec) depend on f

in limit $f \to 0$: (ESS) $\quad p^s_n \to p^{eq}_n = \dfrac{e^{-\beta E_n}}{Z}$

$W_{nm} \to W^{eq}_{nm}$ (equil. trans. matrix)

$$\underbrace{W_{nm}\,p^s_m}_{g(f)} = W^{eq}_{nm}\,p^{eq}_m\left(1 + c_{nm} f + \cdots\cdots\right)$$

coeff. related to Taylor exp: $\dfrac{g'(0)}{g(0)}$

$$g(f) = g(0) + g'(0)f + \frac{1}{2}g''(0)f^2 + \cdots$$

$$= g(0)\left(1 + \frac{g'(0)}{g(0)}f + \frac{1}{2}\frac{g''(0)}{g(0)}f^2 + \cdots\right)$$

plug into avg. $\overline{W}$ expression

$$\overline{W} = -f \sum_{n_1,n_0} \underbrace{W^{eq}_{n_1,n_0}\, p^{eq}_{n_0}}_{\text{symm. under } n_0 \leftrightarrow n_1} \overbrace{\delta x_{n_1,n_0}}^{\text{anti-symm.}} \quad \Big\} \text{ first term } = 0$$

$$- f \sum_{n_1,n_0} W^{eq}_{n_1,n_0}\, p^{eq}_{n_0}\, c_{n_1,n_0}\, f\, \delta x_{n_1,n_0} + \cdots$$

LDB in equil: $\quad W^{eq}_{n_1,n_0}\, p^{eq}_{n_0} = W^{eq}_{n_0,n_1}\, p^{eq}_{n_1}$