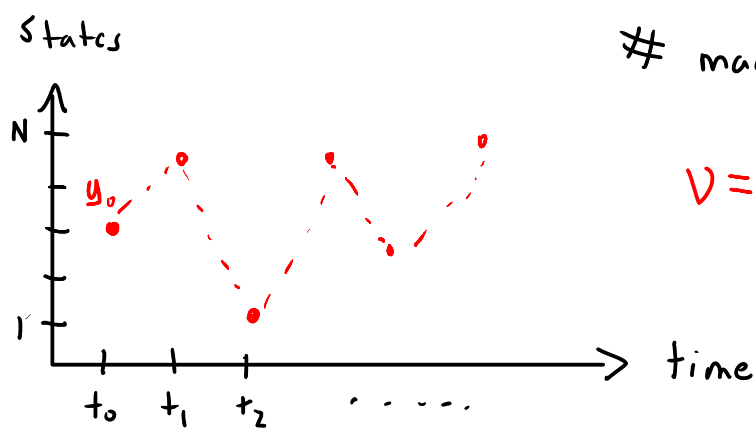
System variable $y(t)$ = labels macrostate of system at time t (could be a vector)

discretize time $\quad t_i = i\,\delta t \qquad i = 0, 1, 2, \ldots$

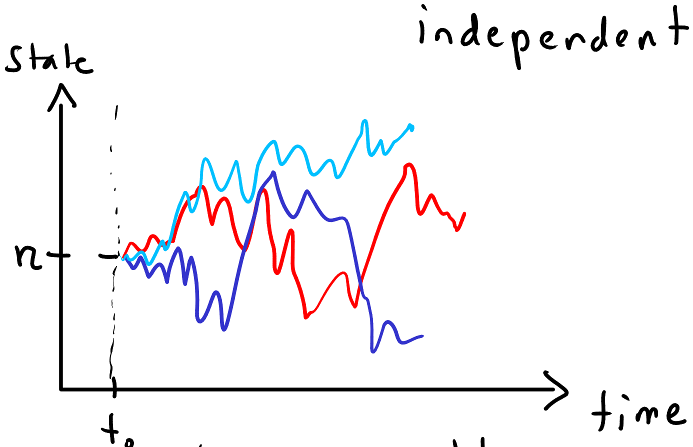
$y(t) \Rightarrow y(t_i) \equiv y_i \Rightarrow$ discretize state: integer labeling macrostate (discrete "addresses")

$1 \le y_i \le N$ = # macrostates

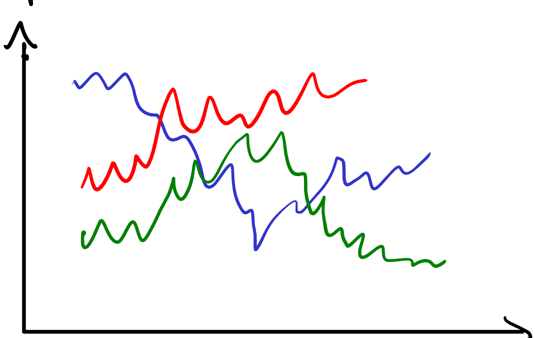


$\nu = (y_0, y_1, y_2, \ldots, y_i)$ = trajectory associated w/ single exper. run

ensemble = collection of traj. from many independent exper. runs



pure ensemble



mixed ensemble

prob. dist. of initial states:

pure: $\quad P(y_0) = \delta_{y_0, n} = \begin{cases} 1 & \text{if } y_0 = n \\ 0 & \text{if } y_0 \neq n \end{cases}$

$P(y_0)$ is not a delta func.

$$\mathcal{P}(\nu) = \mathcal{P}(y_0, y_1, y_2, \ldots, y_i)$$

prob. of observing ν in ensemble

$$= \frac{\#\text{ traj. w/ seq. } (y_0, \ldots, y_i) \text{ in ensemble}}{N_{traj} = \text{tot. \# of traj. (assume large)}}$$

$$\sum_{\nu} \mathcal{P}(\nu) = \sum_{y_0=1}^{N} \sum_{y_1=1}^{N} \cdots \sum_{y_i=1}^{N} \mathcal{P}(y_0, y_1, \ldots, y_i) = 1$$

sum over all possible traj.

physical quantity $Q(\nu)$ assoc. w/ traj. ν

ensemble avg. $\langle Q \rangle = \sum_{\nu} Q(\nu) \mathcal{P}(\nu)$

$Q(\nu) = E_{y_i} - E_{y_0}$ = total energy change

$\langle Q \rangle$ = avg. energy change

Review of basic prob. concepts:

- underlying ensemble
- let A & B are "events" drawn from ensemble

example: $A = y_3$ state at time t_3

<u>or</u> $A = (y_0, y_1, y_2)$

$$\mathcal{P}(A) = \frac{\#\text{ traj. where } A \text{ occurs}}{N_{traj}}$$

joint prob. $$\mathcal{P}(A,B) = \frac{\#\text{ traj. where } A \text{ \underline{and} } B \text{ occur}}{N_{traj}}$$

marginal prob. $$\mathcal{P}(A) = \sum_B \mathcal{P}(A,B)$$

$$\mathcal{P}(B) = \sum_A \mathcal{P}(A,B)$$

all prob's sum to 1: $$\sum_{A,B} \mathcal{P}(A,B) = 1$$

$$\sum_A \mathcal{P}(A) = 1$$

$$\sum_B \mathcal{P}(B) = 1$$

conditional prob: prob. of A <u>given</u> that B occurs

$$\mathcal{P}(A|B) = \frac{\#\text{ traj. where } A \text{ \underline{and} } B \text{ occur}}{\#\text{ traj. where } B \text{ occurs}}$$

$$= \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)}$$

note: $$\sum_A \mathcal{P}(A|B) = \sum_A \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)} = \frac{1}{\mathcal{P}(B)} \sum_A \mathcal{P}(A,B)$$

$\mathcal{P}(A|B)$ is a proper norm. prob. over A

$$= \frac{1}{\mathcal{P}(B)} \mathcal{P}(B) = 1$$

$$\sum_B \mathcal{P}(A|B) = \sum_B \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)} \neq 1 \text{ in general}$$

A and B can be in past / future relative to each other

if A + B are independent: $\mathcal{P}(A,B) = \mathcal{P}(A)\mathcal{P}(B)$

$$\mathcal{P}(A|B) = \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)} = \mathcal{P}(A)$$

$$\mathcal{P}(B|A) = \mathcal{P}(B)$$

last property: Bayes theorem

$$\mathcal{P}(A|B) = \frac{\mathcal{P}(B|A)\,\mathcal{P}(A)}{\mathcal{P}(B)}$$

prove: $\mathcal{P}(B|A) = \frac{\mathcal{P}(A,B)}{\mathcal{P}(A)} \Rightarrow$ plug into RHS

$$\Rightarrow \frac{\mathcal{P}(A,B)}{\mathcal{P}(B)} = \mathcal{P}(A|B)$$

example: 1) statement: I have 3 kids & at least one is a boy.

What is the prob. that I have 3 boys?

$\tilde{B}$ = at least 1 of my kids is a boy

BBB = all three are boys

$$\mathcal{P}(BBB \mid \tilde{B}) = ?$$

all possible traj.

8 traj.

B B B

B B G

B G B

. . .

G G G

$\tilde{B}$ true for 7 out of 8

$$\mathcal{P}(BBB \mid \tilde{B}) = \frac{1}{7}$$

Check Bayes:

$$\mathcal{P}(BBB \mid \tilde{B}) = \frac{\overbrace{\mathcal{P}(\tilde{B} \mid BBB)}^{1}\ \overbrace{\mathcal{P}(BBB)}^{1/8}}{\underbrace{\mathcal{P}(\tilde{B})}_{7/8}}$$

$$= \frac{1}{7}$$

2) "I have 3 kids. Each of them rolled a pair of dice. I have at least one boy who rolled $(1,1)$."

What is the prob. I have 3 boys?