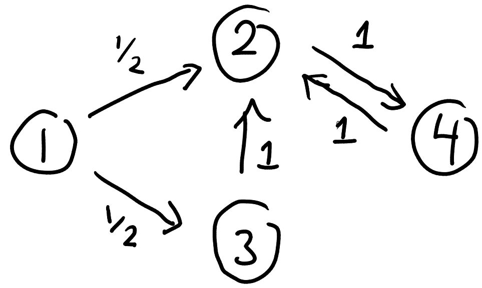
Example: result: ergodic net

$$p_i^s = \frac{1}{h_{ii}}$$

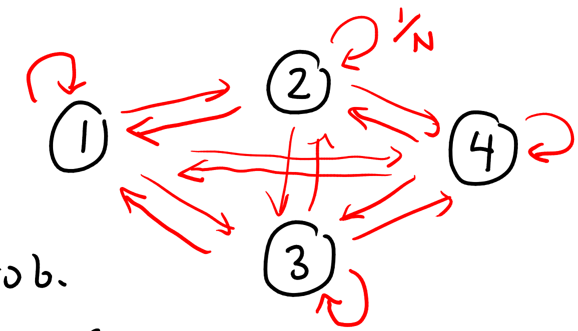
Google Page Rank

$\longrightarrow$ = hyperlink

(i) = webpage



W^H = "trans." matrix created by assigning equal prob. to all outgoing arrows



N = # pages

W^c = trans. matrix of red arrows

$$W^c_{ij} = \frac{1}{N} \text{ for all } i, j$$

$$W = \alpha W^H + \beta W^c$$

if $\alpha + \beta = 1 \Rightarrow$ columns of W sum to 1

$\Rightarrow$ proper trans. matrix

$\beta > 0$: ergodic net

$\alpha = .85$

$\beta = .15$

$\Rightarrow$ calculate $p_j^s = \dfrac{1}{h_{jj}}$ + use it to rank webpages

<u>not</u> using N^2 equ's for h_{ij} to solve this

better approach:

Starting w/ arbitrary $\vec{p}_0$

$$W \cdots\cdots W W \vec{p}^{\,0} \approx \vec{p}^{\,s}$$

$\rightsquigarrow$ final puzzle piece: $W^n \vec{p}^{\,0} \overset{?}{\longrightarrow} \vec{p}^{\,s}$ as $n \to \infty$ for any $\vec{p}_0$?

Next steps: develop a physical framework

classical mechanics for "complex" systems $\Rightarrow$ W matrices

$\swarrow$ MR

$\searrow$ $W^n \vec{p}^{\,0} = \vec{p}^{\,s}$ as $n \to \infty$

Mechanics in d-dimensions w/ M particles

$$\left.\begin{aligned} \vec{q} &= dM \text{ coordinates for all particles} \\ \vec{p} &= dM \text{ momenta " " "} \end{aligned}\right\} 2dM \text{ dim. phase space}$$

exact state of system $\Longleftrightarrow$ $(\vec{q}, \vec{p})$ in this phase space assumptions: system has finite vol. V

no gain/loss of energy from outside

total energy (conserved) $E = \mathcal{H}(\vec{q}, \vec{p})$ Hamiltonian

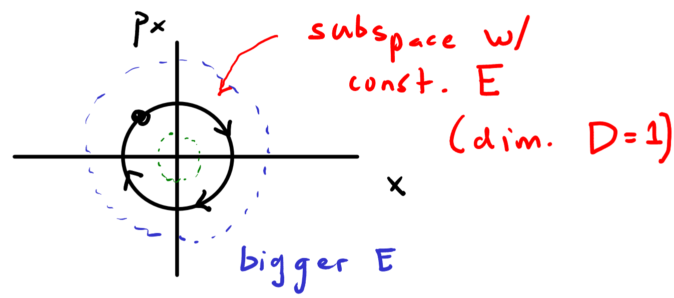
defines a subspace of const. E: is a phase space w/ dim. $2dM-1 \equiv D$

1-dim. harm. oscillator $(m=1)$ $(k=1)$

$$\mathcal{H} = \frac{1}{2} p_x^2 + \frac{1}{2} x^2 = E$$

phase space: (x, p_x)

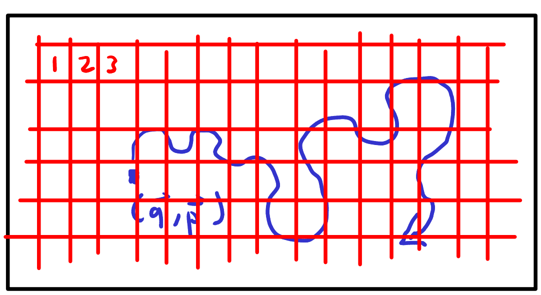
$d=1, \quad M=1$ $\quad 2dM = 2$-dim. phase space



focus: a single subspace (one "layer" of the onion)

$=$ D-dim. surface

cartoon:

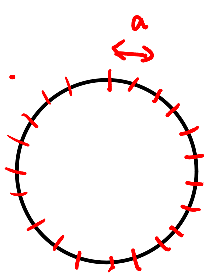


divide up surface into D-dim "boxes" of volume a

label boxes:

$\mu = 1, 2, 3, \ldots\ldots, \Theta_a(E)$

leave out for simplicity



radius $= \sqrt{2E}$

circumf $= 2\pi\sqrt{2E}$

$$\Theta(E) = \frac{2\pi\sqrt{2E}}{a}$$