"Donut" classification of physical systems:

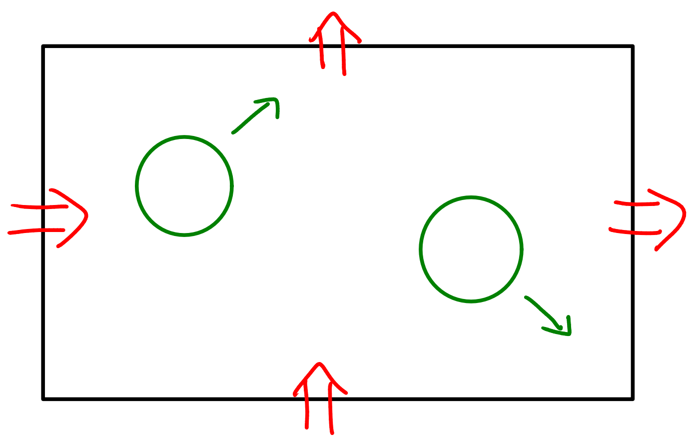
behavior of trajectories	classical version	quantum version
• integrable: confined to donuts	$\mathcal{H} = \mathcal{H}(\vec{I})$	quantization rule $I_K = \hbar(n_K + c_K)$; n_K: integer ≥ 0; c_K: const: Maslov corr.
• non-integrable: confined to deformed donuts	$\mathcal{H} = \mathcal{H}_0(\vec{I}) + \alpha\, U(\vec{\phi}, \vec{I})$, α ↑ small	no simple quantiz. rules
• non-integrable: no donuts! ⇒ chaos	α not small (system-dep.)	

domain of statistical mech.

should lead ~~to~~ ergodicity + mixing (traj. fill out all of phase space)

1970: first rigorous proof that is system could be ergodic + mixing

$\Rightarrow$ Yakov Sinai $\qquad$ 2 · d · N



Sinai billiard

periodic boundary cond.

hard disks colliding

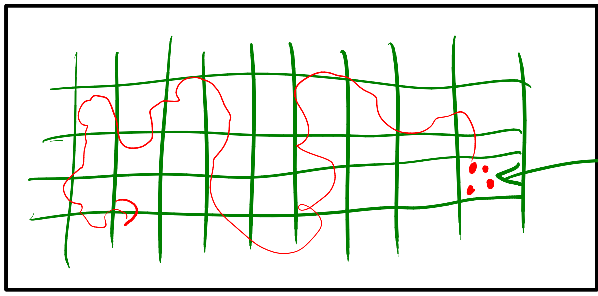
$\Rightarrow$ can prove all possible positions + velocities in phase space visited

current state-of-art:

spheres: $N \geq 2$ in d-dim are almost proven ergodic

rule of thumb: if a system has many particles ("degrees of freedom") + if they are "strongly" interacting

$\Rightarrow$ assume ergodicity + mixing



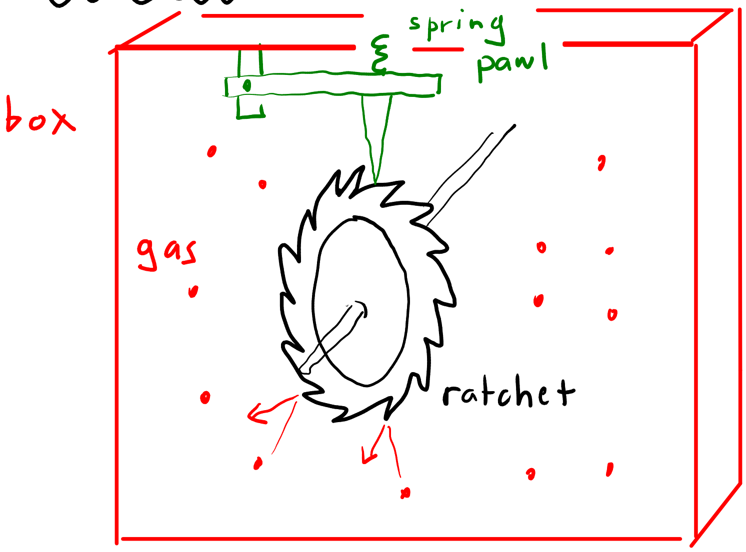
microstate μ

$$P_\mu(t) \xrightarrow[t \to \infty]{}$$

$$P_\mu^S = \frac{1}{\Theta(E)}$$

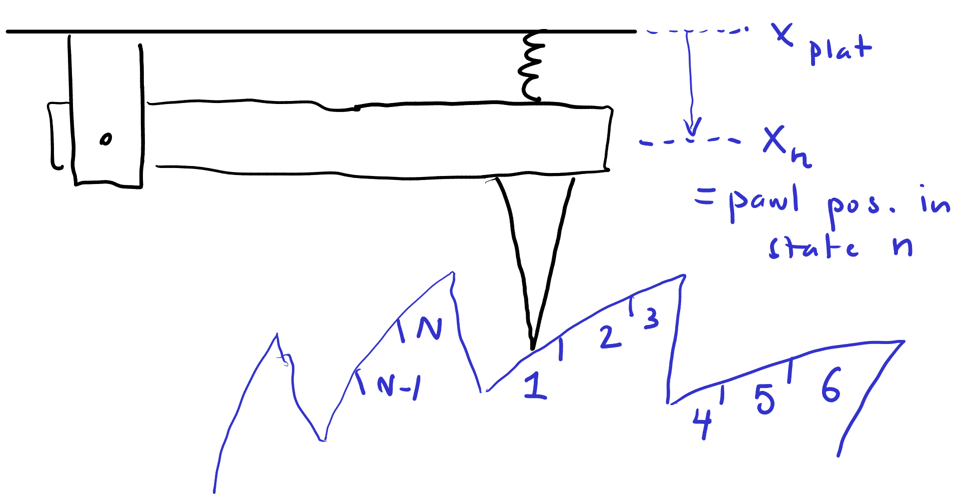
$\Theta(E) = \#$ microstates on surf. w/ total energy E

Example: Feynman's ratchet & pawl



- total system (ratchet, pawl, gas in a box) is isolated
  $\Rightarrow$ total energy is conserved
- assume total system is ergodic & mixing

choose a focus: total = system (ratchet & pawl) + environment (gas in box)



define system states $n = 1, \ldots, N$

E_n = system energy in state n

$$= \underbrace{\tfrac{1}{2}k}_{\text{spring const.}} \left(x_{plat} - x_n - \underbrace{L}_{\text{natural length of spring}} \right)^2$$

$$E_{tot} = E_n + E_n^{env} \qquad E_n^{env} = E_{tot} - E_n$$

E_{tot}: constant (tot. energy conserved)

E_n^{env}: energy of environ. (gas) when system is in state n